Passive array imaging in a waveguide

As for homework 1, we are interested in the case of passive imaging, that is, the array is acting as a receiver and the object that we wish to image as a source. The geometry of the problem is depicted in Figure 1. We dispose of a linear array (in 2d) which records the data

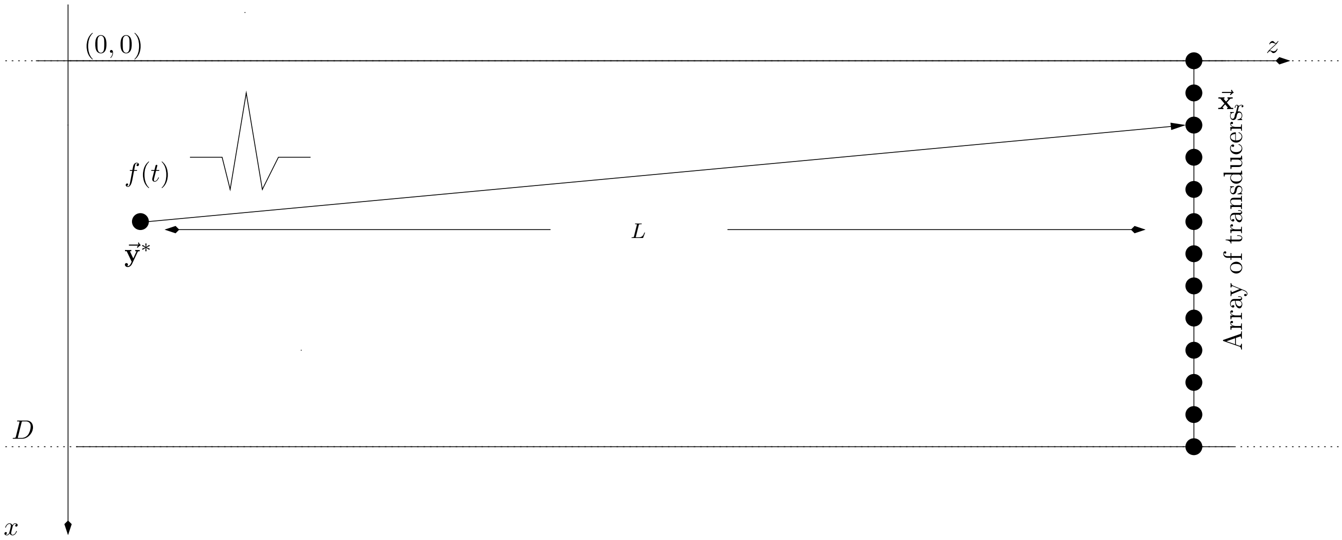

Figure 1: Setup for imaging a distributed source $\mathcal{D}$ with a passive array of transducers in a waveguide.

at the receivers, assumed here to be point transducers located at fixed range $\vec{\mathbf{x}}_r = (x_r, z_r)$, $r = 1, \ldots, N_r$. The aperture of the array is $a = (N_r - 1)\, h$, with h the array pitch, that is, the distance between the receiver elements.

The data recorded at the array is the acoustic pressure field $p(\vec{\mathbf{x}}_r, t)$ solution of the wave equation

$$\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}p(\vec{\mathbf{x}}, t) - \Delta p(\vec{\mathbf{x}}, t) = f(t)\delta(\vec{\mathbf{x}} - \vec{\mathbf{y}}^*), \tag{1}$$

in a waveguide with constant speed of propagation $c_0(\vec{\mathbf{x}}) = c_0 = 1500\text{m/s}$ and depth D. At the boundaries of the waveguide we have the boundary conditions,

$$p(x = 0, z, t) = 0, \quad \frac{\partial}{\partial x}p(x = D, z, t) = 0. \tag{2}$$

In (1) we assume that we have a point source located at $\vec{\mathbf{y}}^* = (x^*, z^*)$. In the project you will have to simulate the presence of more than one sources. In this case, we have

$$\sum_{k=1}^{N_s} \delta(\vec{\mathbf{x}} - \vec{\mathbf{y}}_k^*)$$

instead of $\delta(\vec{\mathbf{x}} - \vec{\mathbf{y}}^*)$ in the second member of (1). Here N_s is the number of point sources and $\vec{\mathbf{y}}_k^*$ their location.

To produce numerically the array data you can use the following expression (for one source)

$$\widehat{p}(\vec{\mathbf{x}}_r, \omega) = \frac{1}{2}\widehat{f}(\omega) \sum_{j=1}^{N(\omega)} \phi_j(x^*)\phi_j(x_r)e^{\imath\beta_j(\omega)(z_r - z^*)} \tag{3}$$

To produce the images, you will use the Kirchhoff migration imaging functional,

$$\mathcal{I}^{\mathrm{KM}}(\vec{\mathbf{y}}^S) = \int d\omega \sum_{r=1}^{N_r} \widehat{p}(\vec{\mathbf{x}}_r, \omega)\overline{G_{WG}(\vec{\mathbf{x}}_r, \vec{\mathbf{y}}^S, \omega)} \tag{4}$$

where

$$\widehat{G}_{WG}(\vec{\mathbf{x}}_r, \vec{\mathbf{y}}^S, \omega) = \frac{1}{2}\widehat{f}(\omega) \sum_{j=1}^{N(\omega)} \phi_j(x^s)\phi_j(x_r)e^{\imath\beta_j(\omega)(z_r - z^s)},$$

is the Green's function in the homogeneous background waveguide. Here

$$\begin{aligned} N(\omega) &= \lfloor \frac{1}{2} + \frac{\omega D}{\pi c_0} \rfloor \\ \phi_j(x) &= \sqrt{\frac{2}{D}} \sin\left(\sqrt{\mu_j} x\right) \\ \mu_j &= \frac{\left(j - \frac{1}{2}\right)^2 \pi^2}{D^2} \end{aligned} \tag{5}$$

Source function For the imaging part assume that the array data are known at the frequency range $[f_0 - B/2, f_0 + B/2]$, with f_0 the central frequency (recall that $\omega = 2\pi f$). For the construction of the data you will need to program the following source function,

$$\widehat{f}(\omega) = e^{-(\omega-\omega_0)^2/(2\sigma^2)}$$

with $\sigma = \pi B/3$.

1. **Full aperture** Consider a waveguide of depth $D = 25$m and a linear array with $N_r = 51$ elements that spans the entire depth of the waveguide. The location of the array elements is $\vec{\mathbf{x}}_r = (x_r, z)$ with $z = L + 10$m and $x_r = (r-1)\ h$, $h = 0.5$ (in m). Take $L = 200$m.

   (a) **One point source** Consider the case of one point source located at $\vec{\mathbf{y}}^* = (12.5, 10)$m.

      i. Construct the KM image using a single frequency $f_0 = 1.5$kHz. What is the resolution in range? in cross-range ?

ii. Use as bandwidth $B = 1$kHz, what happens?

iii. Using a fixed frequency $f_0 = 1.5$kHz, move the source location in cross-range, what do you observe? That is move x^* from 0 to D, what happens to the image? How far should the source be from the boundary to be seen?

iv. Using a fixed frequency $f_0 = 1.5$kHz and for a source located at $\vec{\mathbf{y}}^* = (12.5, 10)$m double the range, i.e., take $L = 400$m what happens to the image?

(b) **Two point sources**

i. How far should two sources be in range so that we can see them as separate objects? put two sources at this distance and construct the corresponding image.

ii. How far should two sources be in cross-range so that we can see them as separate objects? put two sources at this distance and construct the corresponding image.

iii. Consider the same configuration in free space, what do you observe? That is, take the same array size, same f_0, B and L.

2. **Partial aperture** Consider now a smaller array composed by $N_r = 26$ receivers. Take $\vec{\mathbf{x}}_r = (x_r, z)$ with $z = L + 10$m and $x_r = 7.25 + (r - 1)\ h$, $h = 0.5$ (in m), $L = 200$m. Take first one point source located at $\vec{\mathbf{y}}^* = (12.5, 10)$m.

(a) Construct the KM image using a single frequency $f_0 = 1$kHz. What is the resolution in range? in cross-range ?

(b) Use as bandwidth $B = 1$kHz, what happens?

(c) Using a fixed frequency $f_0 = 1$kHz, move the source location in cross-range, what do you observe? That is, move x^* from 0 to D, what happens to the image? How far should the source be from the boundary to be seen?

(d) Using a fixed frequency $f_0 = 1$kHz and for a source located at $\vec{\mathbf{y}}^* = (12.5, 10)$m double the range, i.e., take $L = 400$m what happens to the image?

(e) How do the above results compare with the full aperture case?

All images should be computed on a square of size 25×25 m with a discretization of 0.25m and centered at $\vec{\mathbf{y}}^*$.