Passive array imaging in free space

In the case of passive imaging the array is acting as a receiver and the object that we wish to image as a source. The geometry of the problem is depicted in Figure 1. We dispose

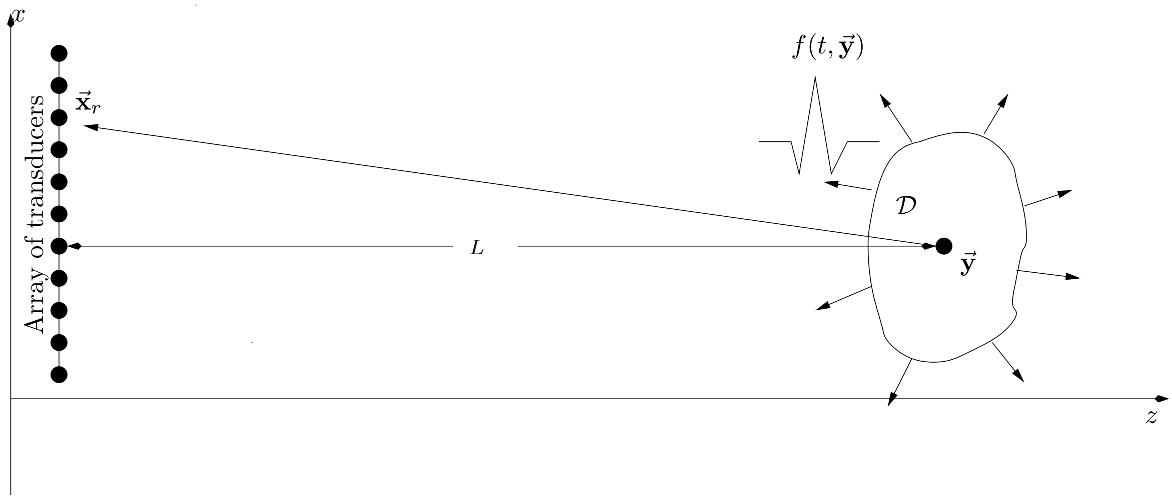

Figure 1: Setup for imaging a distributed source $\mathcal{D}$ with a passive array of transducers in free space.

of a linear array (in 2d) which records the data at the receivers, assumed here to be point transducers located at $\vec{\mathbf{x}}_r$, $r = 1, \ldots, N$. The aperture of the array is $a = (N-1)\,h$, with h the array pitch, that is, the distance between the receiver elements. The data recorded at the array is the acoustic pressure field $p(\vec{\mathbf{x}}_r, t)$, with $p(\vec{\mathbf{x}}, t)$ the solution of the wave equation,

$$\frac{1}{c_0^2}\frac{\partial^2}{\partial t^2}p(\vec{\mathbf{x}}, t) - \Delta p(\vec{\mathbf{x}}, t) = f(\vec{\mathbf{x}}, t). \tag{1}$$

In (1) we assume that the source function $f(\vec{\mathbf{x}}, t)$ has compact support over the domain D. In imaging we are interested in solving the following problem:

Problem 1 *Find the support domain D of the source, given the array data $p(\vec{\mathbf{x}}_r, t)$ in some time interval $[0, T]$.*

We will assume that the source function $f(\vec{\mathbf{x}}, t)$ is of the following form,

$$f(\vec{\mathbf{x}}, t) = f(t)\mathbb{1}_D(\vec{\mathbf{x}}), \tag{2}$$

where $\mathbb{1}_D(\vec{\mathbf{x}})$ is the indicator function of the domain D.

Equivalently, in the frequency domain $\widehat{p}(\vec{\mathbf{x}}, \omega)$, the fourier transform of $p(\vec{\mathbf{x}}, t)$, is the solution of the reduced wave (Helmholtz) equation

$$\Delta \widehat{p}(\vec{\mathbf{x}}, \omega) + \frac{\omega^2}{c_0^2} p(\vec{\mathbf{x}}, \omega) = -\widehat{f}(\omega) \mathbb{1}_D(\vec{\mathbf{x}}) \tag{3}$$

The imaging problem becomes,

Problem 2 *Find the support domain D of the source, given the array data $\widehat{p}(\vec{\mathbf{x}}_r, \omega)$ in some frequency interval $[\omega_{min}, \omega_{max}]$.*

Remark 1 *To the wave equation (1) we should add zero initial conditions for $p(\vec{\mathbf{x}}, t)$ and $\partial_t p(\vec{\mathbf{x}}, t)$.*

Remark 2 *The wave equations (1) and (3) are a priori posed in the whole space (here $\mathbb{R}^2$) and we assume the Sommerfeld radiation condition at infinity. When solving these equations numerically adequate absorbing boundary conditions should be used at the boundaries of the computational domain (I suggest PML).*

Remark 3 *In the wave equations (1) and (3), the wave propagation velocity depends, in general, on position. For this project we assume that it is constant $c_0(\vec{\mathbf{x}}) = c_0 = 1500 m/s$.*

To produce numerically the array data we can solve the wave equation either in the time or in the frequency domain. We can also use the following expression

$$\widehat{p}(\vec{\mathbf{x}}_r, \omega) = \widehat{f}(\omega) \int_D d\vec{\mathbf{y}} \widehat{G}_0(\vec{\mathbf{x}}_r, \vec{\mathbf{y}}, \omega), \tag{4}$$

with

$$\widehat{G}_0(\vec{\mathbf{x}}, \vec{\mathbf{y}}, \omega) = \frac{e^{i\,\omega \frac{|\vec{\mathbf{x}} - \vec{\mathbf{y}}|}{c_0}}}{4\pi\, |\vec{\mathbf{x}} - \vec{\mathbf{y}}|},$$

being the Green's function in the homogeneous background medium.

To produce the images, you should use the Kirchhoff migration imaging functional,

$$\mathcal{I}^{\mathrm{KM}}(\vec{\mathbf{y}}^S) = \int d\omega \sum_{r=1}^{N} \widehat{p}(\vec{\mathbf{x}}_r, \omega) \overline{G_0(\vec{\mathbf{x}}_r, \vec{\mathbf{y}}^S, \omega)} \tag{5}$$

Source function For the imaging part assume that the array data are known at the frequency range $[f_0 - B/2, f_0 + B/2]$, with f_0 the central frequency (recall that $\omega = 2\pi f$). For the construction of the data you will need to program the following two source functions,

$$\widehat{f}_1(\omega) = \mathbb{1}_{[\omega_0 - \pi B, \omega_0 + \pi B]}$$

or

$$\widehat{f_2}(\omega) = e^{-(\omega-\omega_0)^2/(2\sigma^2)}$$

with $\sigma = \pi B/3$.

1. **Linear array** Consider a linear array with $N = 51$ elements and array pitch $h = 0.5$m. The location of the array elements is $\vec{\mathbf{x}}_r = (x_r, z)$ with $z = 10$m and $x_r = 10 + (r-1)\,h$ (in m). We call the direction x the cross-range and z the range.

   (a) **Point source** Consider the case of a point source located at $\vec{\mathbf{y}}^* = (22.5, L+10)$m, $L = 200$m.

      i. Construct the KM image using only one frequency, $f_0 = 1.5$kHz. What do you observe? What is the resolution in range? in cross range? Compare with the theory. To see better the resolution in range you can plot the image as a function of range at the correct cross-range of the source. The same can be done with the cross-range. Increase the frequency to $f_0 = 3$kHz. What do you observe?

      ii. Construct the KM image using $f_0 = 1.5$kHz and $B = 1$kHz. What do you observe? What is the resolution in range? in cross range? Compare with the theory. Now take, $f_0 = 3$kHz and $B = 1$kHz what do you observe? Increase the bandwidth to $B = 3$kHz, what do you observe? Compare with the theory.

      iii. Compare the results using the source function $f_1(\omega)$ and $f_2(\omega)$, what do you observe?

   From now on, use only $f_2(\omega)$.

   (b) **Extended source** Consider the case of an extended source. Take for example, as domain D a square with center $\vec{\mathbf{y}}^* = (22.5, L+10)$m, $L = 200$m and size $b = 16$m.

      i. Construct the KM image using only one frequency, $f_0 = 1.5$kHz. What do you observe?

      ii. Construct the KM image using $f_0 = 1.5$kHz and $B = 1$kHz. What do you observe?

      iii. What happens for a smaller object of size $b = 4$m?

2. **Circular array** Consider a circular array with $N = 101$ (equidistant) elements. The location of the array elements is on a circle centered at zero with radius $r = 20$m. You might need to increase the number of array elements if the results are not very good, make some tests to decide.

   (a) **Point source** Consider the case of a point source located at $\vec{\mathbf{y}}^* = (0, 0)$m.

      i. Construct the KM image using only one frequency $f_0 = 1.5$kHz and $f_0 = 3$kHz. What do you observe?

ii. Construct the KM image using $f_0 = 3$kHz and $B = 1$kHz what do you observe? Increase the bandwidth to $B = 3$kHz, what do you observe?

(b) **Extended source** Consider the case of an extended source. Take for example, as domain D a square with center $\vec{\mathbf{y}}^* = (0, 0)$m and size $b = 4$m what do you observe?

All images should be computed on a square of size 25×25 m with a discretization of 0.25m and centered at $\vec{\mathbf{y}}^*$.