Physica Scripta. Vol. T84, 18–21, 2000

Stabilizing the Pereira-Stenflo Solitons in Nonlinear Optical Fibers

Nikos Efremidis[1], Kyriakos Hizanidis[1], Boris A. Malomed[2], Hector E. Nistazakis[3] and Dimitri J. Frantzeskakis[3]

[1]Department of Electrical and Computer Engineering, National Technical University of Athens, 9 Iroon Polytexniou, 157 73 Athens, Greece
[2]Department of Interdisciplinary Studies, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel
[3]Department of Physics, University of Athens, Panepistemiopolis, 157 84 Athens, Greece

Reeived September 6, 1999

PACS Ref: 42.81.Dp, 41.20.Jb, 42.81.Wg

Abstract

We study in detail stability of *exact* chirped solitary-pulse solutions in a model of a filtered nonlinear optical fiber, in which stabilization of the pulses is achieved by means of an extra lossy core, parallel-coupled to the main one. We demonstrate that, in the model's three-dimensional parameter space, there is a vast region where the pulses are fully stable, for both signs of the group-velocity dispersion. These results open way to a stable transmission of optical solitons in the *normal*-dispersion region and, thus, to an essential expansion of the bandwidth offered by the nonlinear optical fibers for telecommunications. In the cases when the pulses are unstable, we study the development of the instability, which may end up by either a blowup or decay to zero.

1. Introduction

It is commonly known that solitons may exist in optical fibers in regions of anomalous dispersion [1]. The losses, which are inevitably present in the fibers, can be compensated by Er-doped amplifiers [2]. However, the amplifiers give rise to various instabilities and related detrimental effects, the most dangerous one being a random jitter of the solitons induced by their interaction with an optical noise which is generated by the amplifiers [3]. Schemes providing for stabilization of the transmission of periodically amplified solitons have been developed, based, first of all, on guiding filters [4].

A basic model that takes into regard the group-velocity dispersion, Kerr's nonlinearity, amplification, and fixed-frequency guiding filters is a perturbed nonlinear Schrödinger (NLS) equation [3]:

$$\mathrm{i}u_z + (1/2)Du_{tt} + |u|^2u = \mathrm{i}u + \mathrm{i}u_{tt}\,, \tag{1}$$

where z and t are the propagation distance and so-called reduced time, and D is the dispersion coefficient. The nonlinearity coefficient, excessive gain, and an effective filtering strength are all normalized to be $\equiv 1$. The gain and filtering terms in Eq. (1) are assumed to be uniformly distributed along the long fiber link. This approximation, that disregards the discrete character of the amplification and filtering, is well justified for the propagation of sufficiently broad solitons (with the temporal width $\tau \gtrsim 10$ ps), whose soliton period is essentially larger than the amplification spacing [3].

Up to a change of the notation, Eq. (1) is, simultaneously, the complex cubic Ginzburg-Landau (CCGL) equation, which is a well-known paradigm model in the nonlinear pattern-formation theory [5]. It is well known that the CCGL equation (1) has an exact pulse solution, or the *Pereira-Stenflo* (PS) soliton [6], which describes a stationary bright pulse with an internal *chirp*. It is necessary to stress that, while the unperturbed NLS equation, i.e., Eq. (1) without the right-hand side, has (bright) soliton solutions only in the case of anomalous dispersion, $D > 0$, the full CCGL equation (1) has exact pulse solutions in all the cases (also when the coefficient in front of the cubic term is complex, taking into regard, in terms of the nonlinear optics, two-photon absorption). An explanation to this property is that, in the unperturbed NLS equation, the compensation between the nonlinearity and dispersion is only possible when the dispersion is anomalous, while in the CCGL equation the nonlinearity, dispersion, and filtering may all be in balance through the internal chirp of the stationary soliton.

A fundamental drawback of the exact PS solution to Eq. (1) is that it is *unstable*, as the zero solution to this equation, i.e., the soliton background, is unstable against perturbations $\sim \exp(-\mathrm{i}\omega t)$ with $\omega^2 < 1$. This instability reflects a fundamental problem which exists in systems in which the transmission of solitons is supported by means of a distributed linear gain. However, the rate at which the instability grows and, eventually, destroys the pulse strongly depends (for a fixed level of initial small perturbations that are amplified by the instability) on the relation between the dispersion and filtering, i.e., on the value and sign of the coefficient D in Eq. (1). Recently, this issue was investigated in detail by means of direct numerical simulations in Ref. [7]. It was demonstrated that, in the case of *normal* dispersion, $D < 0$, the distance z_{stab} of the stable propagation (followed by an instability-induced blowup) is much larger (by a factor of up to ~ 100) than in the anomalous-dispersion case. The distance z_{stab} reaches a well-pronounced maximum at D close to its *optimum value*, $D_{\mathrm{opt}} \approx -18$, and then gradually decreases with the further increase of $|D|$ inside the normal-dispersion region. Thus, quasi-stable transmission of pulses in the *normal-dispersion* range of the carrier wavelengths is possible, and may be actually more stable than in the anomalous-dispersion range.

Nevertheless, full stability of the pulses cannot be achieved within the framework of the model (1). A new approach allowing one to suppress the instability of the zero state and thus open way to generation of *absolutely stable* solitons was proposed in Ref. [8] and then checked by means of direct simulations in Ref. [9]: one should linearly couple the fiber to an additional parallel lossy fiber. It should be stressed that there is no need to actually replace the usual single-core communication fiber by a dual-core one. In reality, because the coupling length between the cores is very short, typically $\sim$ several cm, one can add short segments of the parallel lossy core, integrating them into one unit with the periodically spaced amplifiers and filters. As well as the gain and filtering, this new ingredient can be then assumed to be uniformly distributed along the communication line.

The system with the extra lossy core is *bistable*: it has stable states in the form of the zero solution and nontrivial soliton with uniquely determined parameters, and, additionally, an unstable soliton with its amplitude smaller and width larger than those of the stable one. Later, it was demonstrated [10] that the stable soliton in this dual-core model can be found in an *exact* form. However, the numerical stability analysis of the pulses performed in Ref. [10] was sketchy and comprised limited parametric regions. In this work, we aim to develop a systematic analysis of the pulse transmission in the stabilized model. We demonstrate that absolutely stable pulses can propagate at both normal and anomalous values of the dispersion, which opens way for a very efficient use of the fiber's bandwidth.

The rest of the paper is organized as follows. In Section 2, we formulate the model, giving estimates for the values of its parameters, and describe its exact solitary-pulse solution of the PS type. In Section 3, we perform a systematic analysis of the stability of the zero solution in it, which provides for a necessary basis for the direct numerical analysis of the soliton's stability in Section 4. The main result of Section 4 is a three-dimensional picture that shows a full stability region in the model's parameter's space. Section 5 (in which we also briefly mention results obtained for interaction between two stable pulses in the present model) concludes the paper.

2. The model

Two parallel-coupled optical fibers are described by a system of the following form [10] (cf. Eq. (1)):

$$\mathrm{i}u_z + (D/2 - \mathrm{i})u_{tt} + |u|^2 u - \mathrm{i}u = Kv, \tag{2}$$

$$\mathrm{i}v_z + \mathrm{i}\Gamma v + k_0 v = Ku. \tag{3}$$

Here, v is the field amplitude in the extra core with a loss constant Γ in it, K is a coupling constant, k_0 being a phase-velocity mismatch between the two cores. As it is demonstrated in [9] and [10], the nonlinearity and dispersion in the additional core may be neglected, and the filtering inside it, as well a possible group-velocity mismatch between the cores, also produce inconsiderable effects and are therefore neglected.

To estimate relevant values of the renormalized parameters kept in the model, we recall that the gain bandwidth of the Erbium-doped optical amplifier, which produces the gain, is $\Delta\omega \sim 10^{12}$ Hz [2], which can be reduced by the filters to $\Delta\omega \sim 10^{11}$ Hz. A typical value for the fiber losses is $\gamma \approx 0.005$ km^{-1}, so that the filtering coefficient $\gamma_2 = \gamma/(\Delta\omega)^2$ takes values in an interval $0.005 \div 5$ ps^2/km. A typical value of the physical dispersion coefficient in the telecommunication fibers is $|\beta_2| = 20$ ps^2/km [1], so that our dimensionless dispersion coefficient $D \equiv \beta_2/\gamma_2$ takes values within a broad interval $|D| \leq 400$.

The system of Eqs. (2) and (3) possesses an exact analytical solution [10], which follows the pattern of the original PS soliton [6]:

$$u = u_0 \mathrm{e}^{ikz}[\mathrm{sech}(\eta t)]^{(1+\mathrm{i}\mu)}, \; v = v_0 \mathrm{e}^{ikz}[\mathrm{sech}(\eta t)]^{(1+\mathrm{i}\mu)}, \tag{4}$$

where the "chirp" parameter is $\mu = -(3/4)D + (1/4)\sqrt{32 + 9D^2}$, the amplitudes u_0 and v_0 in the gain- and loss-cores are linearly related, $v_0 = (k_0 - k + \mathrm{i}\Gamma)^{-1} K u_0$, while the remaining parameters u_0, η and k are determined as follows. Firstly, the relative phase velocity mismatch $\delta = k_0 - k$ is calculated by means of a cubic equation,

$$\begin{aligned} &-(\mu D - 2)\delta^3 + \left[(1-\mu^2)DD + 4\mu + k_0(\mu D - 2)\right]\delta^2 + \\ &+(\mu D - 2)(K^2 - \Gamma^2)\delta + \left[(1-\mu^2)D + 4\mu\right]\Gamma(\Gamma - K^2) + \\ &\quad (\mu D - 2)k_0\Gamma^2 = 0. \end{aligned} \tag{5}$$

Secondly, the soliton's inverse width η is given by

$$\eta^2 = \frac{\delta(1-\Gamma) + k_0\Gamma}{\delta(2 - D\mu) + \Gamma[D(1-\mu^2) + 4\mu]}, \tag{6}$$

and, finally, the amplitude u_0 is determined by the expression $u_0^2 = (3/4)\mu\left(4 + D^2\right)\eta^2$. Physical solutions to the cubic equation (5) are those which yield $\eta^2 > 0$.

Thus, an exact analytical solution in the form of a chirped solitary pulse is available in the present model. We will see in the following sections that stability analysis for this solution is a fairly complicated problem, which can be solved only numerically.

3. Stability of the zero solution

Proceeding to the stability analysis, we first of all notice that the solitary pulse (4) cannot be stable unless its background, i.e., the zero solution, $u = 0$, $v = 0$, is stable. Recall that it is exactly the instability of the zero solution which renders unstable the usual PS solution to the CCGL equation (1).

In order to investigate the stability of the zero solution, we linearize Eqs. (2)–(3), substituting into them infinitesimal perturbations,

$$u = u_1 \mathrm{e}^{\mathrm{i}(kz - \omega t)}, \quad v = v_1 \mathrm{e}^{\mathrm{i}(kz - \omega t)}, \tag{7}$$

where k and ω are the (generally, complex) wavenumber and (always real) frequency of the perturbations, respectively. Then, the stability region in the plane of the model's parameters (Γ, K) is determined by the condition $\mathrm{Im}\,k > 0$, which, after some algebra, leads an inequality

$$K^2 > \Gamma(1 - \omega^2)\left[1 + \frac{\left(2k_0 + D\omega^2\right)^2}{4(\Gamma - 1 + \omega^2)^2}\right]. \tag{8}$$

This inequality holds automatically, provided that $\omega^2 > 1$. Therefore, we need to consider Eq. (8) only for $\omega^2 < 1$. Additionally, it is readily seen that the condition $\Gamma > 1$ must hold for the right-hand side of Eq. (8) to remain finite at these values of ω. In the case $\omega^2 < 1$, Eq. (8) can be simplified in some special cases, e.g., $k_0 = D = 0$, or $\Gamma >> 1$, or $k_0 = \omega = 0$, yielding, in all these cases,

$$1 < \Gamma < K^2. \tag{9}$$

This is a known condition [8–10] which defines an area in the

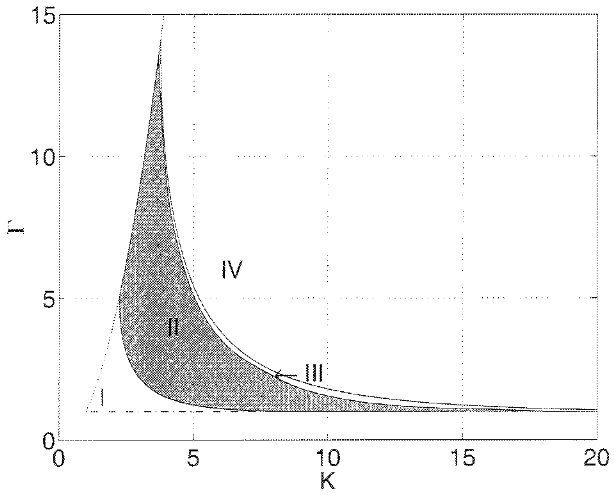

Fig. 1. Regions of stability of the zero solution and regions of existence and stability of the exact solitary-wave solution in the (Γ, K) parameter plane in the dual-core model with $D = -18$ and $k_0 = 0$. Region I: the zero solution is unstable. Region II: The solitary-wave solution is stable. Region III: the zero solution is stable, while the solitary-wave solution is not, showing a decay into zero. Region IV: the solitary-wave solution does not exist. In the region located outside the region I, which is defined by the curves $\Gamma = K^2$ (dotted curve) and $\Gamma = 1$ (dotted-dashed curve), the zero solution is unstable according to Eq. (9).

(K, Γ) plane, limited by two curves shown in Fig. 1, $\Gamma = K^2$ and $\Gamma = 1$, where the zero solution is stable in the above-mentioned special cases.

In what follows below, we will, chiefly, concentrate on the most straightforward case $k_0 = 0$, although changes brought about by $k_0 \neq 0$ will also be considered. Setting $k_0 = 0$, we also fix D, aiming to analyze the stability condition (8) in the (Γ, K) plane. As for the choice of the value of D, it was mentioned above [7] that, in the case of the single CCGL equation (1), the propagation distance z_{stab} before the onset of the soliton's instability takes its maximum value around $D = -18$, hence it seems natural to dwell, first of all, on this value.

Upon investigating the condition (8) numerically, we have found, varying the arbitrary real frequency ω, that there exists a hyperbola-like curve, which further reduces the stability region of the zero solution defined by Eq. (9) (that is, the region I is excluded), as is shown in Fig. 1. It is noticed that in the case $\Gamma \gg 1$, this numerically found curve coincides with the one $\Gamma = K^2$, while for large values of K, it asymptotically approaches the straight line $\Gamma = 1$.

4. The stability of the solitons

Proceeding from the stability conditions for the zero solutions to the full stability analysis for the PS-like solutions given by Eq. (4), one should, first of all, isolate a region in the (Γ, K) plane where these solutions actually exist. The existence condition may be readily found upon utilizing Eq. (5), which may have one or three real solutions for k, that determines the soliton's inverse width η [see Eq. (6)]. Evidently, the condition $\eta^2 > 0$ selects physical solutions of the cubic equation (5). Following this way, we have numerically found that there is an additional hyperbola-like curve (a thick solid one in Fig. 1) in the (Γ, K) plane, below which the existence of at least one real positive value of η^2 is guaranteed. Beyond this curve, (in region IV, Fig. 1), the solutions given by Eq. (4) do not exist.

Thus, we have found a finite, boomerang-shaped region in the (Γ, K) plane (regions II and III, Fig. 1), where the two conditions, viz., the stability of the zero solution and the existence of the exact solitary-wave one, are simultaneously satisfied. However, direct numerical simulations of the solitary-pulse's stability demonstrate that it is really stable only in the shaded portion (region II, in Fig. 1; recall, we are now dealing with the case $D = -18$ and $k_0 = 0$). On the contrary, in the slim region III the solitary solutions exist but are unstable. Inside the region II, the soliton is found to be completely stable over indefinitely long evolution.

Inside region I, where the zero solution is unstable, direct simulations show that the evolution of the pulse ends by a blowup. Comparison with the results of Ref. [7] demonstrates that the propagation distance z_{stab} before the onset of the blowup in the present dual-core model is about *an order of magnitude* larger than in the single-core model at the same values of the parameters [7]. The further one moves away from the region II (deeper into the region I) the wider the range of frequencies that generate the instability becomes, resulting in a decrease of the propagation distance. The simulations also show that, inside the region III, the soliton propagates, initially, stably, but then it rapidly decays to zero (rather than blowing up), in compliance with the fact that the zero solution is stable in this region.

Coming back to the four regions distinguished in Fig. 1, we conclude that, inside the region I, the zero and solitary-wave solutions are both unstable. Then, as K is increased, we cross into region II, where there coexist stable and unstable solitary-wave solutions, the zero solution being stable. In this region, the stable soliton, along with the stable zero background, act as attractors for the unstable solitary wave solution: simulations demonstrate that there is a subregion II(a), in which an initial configuration in the form of the unstable pulse decays into the stable trivial solution, while, in another subregion II(b), it evolves into the stable pulse. Lastly, in region III, both solitary-wave solutions are unstable and collapse into the zero solution.

So far, we analyzed the stability of the solitary wave for fixed values $D = -18$ and $k_0 = 0$. In order to investigate the change of the stability region with varying D and k_0, we note at first that the stability region II of the solitary wave almost covers the region of stability of the zero solution (regions II and III). Thus we may evaluate numerically the area of the stability region for the solitary wave, upon determining the boundary curves separating the regions I, II and III, IV. The results are shown in Fig. 2, where the stability region's area is plotted *vs.* D for $k_0 = -1, 0, 1/2, 1$. As it can be seen, in the case $k_0 = 0$ the stability area is an even, parabola-like function of D that increases with $|D|$. As ones increases k_0 to positive values ($k_0 = 0.5$ or 1), the stability area monotonically decreases, creating an interval around $D = 0$ where *all* the solutions are unstable. On the other hand, if we decrease k_0 to negative values, the stability area does not decrease monotonously. In particular, for $k_0 = -1$, it is observed that, for $D \gtrsim 10$, the stability area is increased, while for negative values $D \lesssim -7$, it almost coincides with that corresponding to $k_0 = 0$.

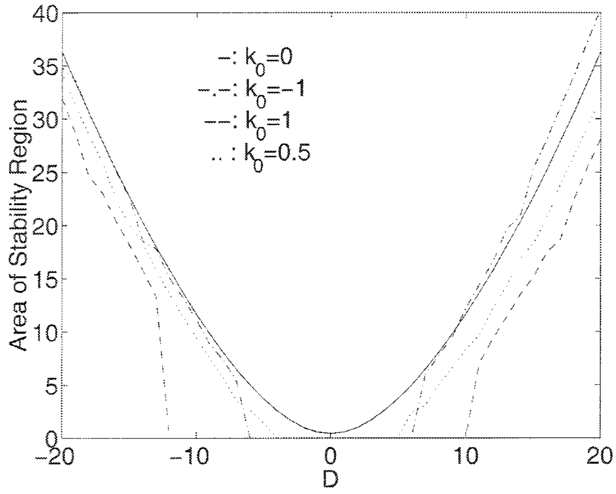

Fig. 2. The area of the stability region of the solitary-wave solution vs. D for $k_0 = -1, 0, 0.5, 1$.

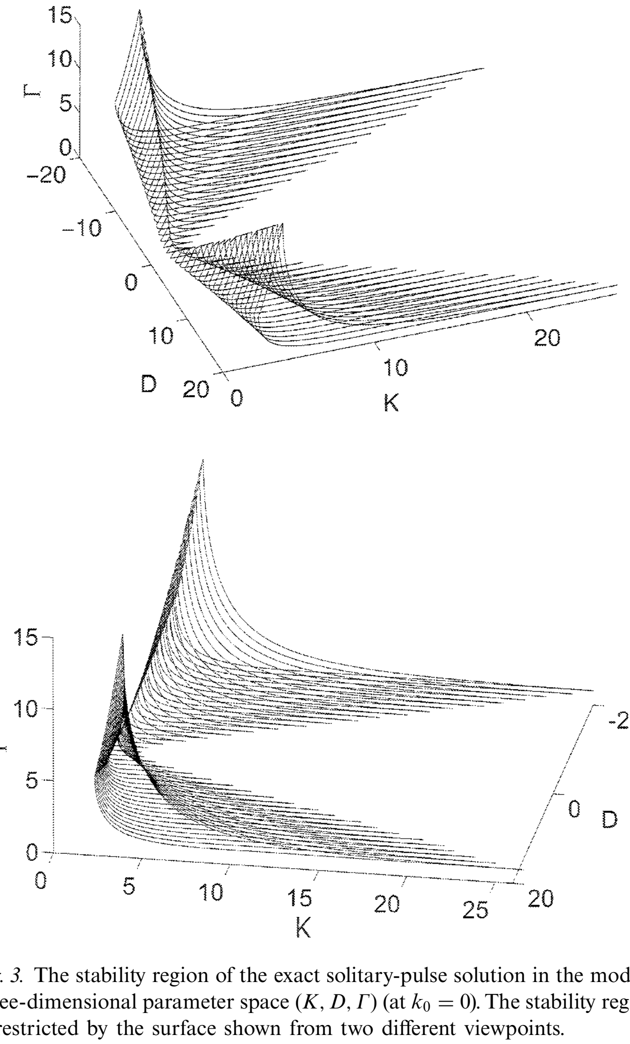

Fig. 3. The stability region of the exact solitary-pulse solution in the model's three-dimensional parameter space (K, D, Γ) (at $k_0 = 0$). The stability region is restricted by the surface shown from two different viewpoints.

Coming back to the most essential case $k_0 = 0$, we can collect all the data concerning the stability of the exact solitary-pulse solutions in the form of a three-dimensional picture displayed in Fig. 3. This figure shows the stability region in the parameter space (K, D, Γ) from two different directions in this space. This result, showing fairly large stability regions at positive and negative D (i.e., anomalous and normal group-velocity dispersion in the main core) is the main result of this work.

5. Conclusion

In this work, we have studied in detail stability of *exact* chirped solitary-pulse solutions in a model in which stabilization of the pulses in a nonlinear optical fiber with guiding filters is achieved by means of short segments of an additional lossy core which is parallel-coupled to the main one. The exact solutions follow the pattern of the well-known Pereira-Stenflo solitons. We have demonstrated that, in the model's three-dimensional parameter space, there is a vast region where the pulses are fully stable, for both signs of the group-velocity dispersion, normal and anomalous. These results open way to a stable transmission of optical solitons in the *normal*-dispersion region and, thus, to an essential expansion of the bandwidth offered by the nonlinear optical fibers for telecommunications. In the cases when the pulses are unstable, we have studied in detail the development of the instability, which may end up by either a blowup or collapse to zero.

In addition to the results displayed in detail in this paper, we have also simulated interactions between two stable pulses, placed, initially, at some distance from each other. The initial pulses are identical, the only difference between them being a phase shift. A result is that, irrespective of the phase shift, the pulses *always* attract each other and, ultimately, merge into a new single pulse, which is identical to each of the two initial ones.

References

1. Agrawal, G. P., "Nonlinear Fiber Optics" (Academic Press: San Diego, 1995).
2. Desurvire, E., "Erbium-Doped Fiber Amplifiers" (John Wiley & Sons, New York, 1994).
3. Hasegawa, A. and Kodama, Y., "Solitons in Optical Communications" (Oxford University Press: Oxford 1995).
4. Mecozzi, A., Moores, J. D., Haus, H.A. and Lai, Y., Opt. Lett. **16**, 1841 (1991); Kodama, Y. and Hasegawa, A., Opt. Lett. **17**, 31 (1992).
5. Cross, M. C. and Hohenberg, P. C., Rev. Mod. Phys. **65**, 851 (1993).
6. Pereira, N. R. and Stenflo, L., Phys. Fluids **20**, 1733 (1977); Hocking, L. M. and Stewartson, K., Proc. Roy. Soc. London Ser. A **326**, 289 (1972).
7. Malomed, B. A., Frantzeskakis, D. J., Nistazakis, H. E., Tsigopoulos, A. and Hizanidis, K., Phys. Rev. E, **60**, 3324 (1999).
8. Malomed, B. A. and Winful, H. G., Phys. Rev. E **53**, 5365 (1996).
9. Atai, J. and Malomed, B. A., Phys. Rev. E **54**, 4371 (1996).
10. Atai, J. and Malomed, B. A., Phys. Lett. A **246**, 412 (1998).

