Supplemental material for "Observation and optical tailoring of photonic lattice filaments"

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Numerical model.— According to our previous theoretical work [1], the lattice filament does not exhibit strong space-time coupling effects [2], allowing us to omit dispersion and investigate here the spatial properties of the lattice filament. The numerical model that is used in the simulations resolves a nonlinear Schrödinger type equation:

$$\frac{\partial \mathcal{E}}{\partial z} = \frac{i}{2k_0}\Delta_\perp \mathcal{E} + N(\mathcal{E}^2,\rho)\mathcal{E} + ik_0\Delta n(x,y)\mathcal{E} \tag{1}$$

which describes the evolution of the slowly varying envelope $\mathcal{E}(x,y,z,t)$ of the electric field $E = \mathbb{R}[\mathcal{E}\exp(ik_0z - i\omega_0 t)]$ of a laser pulse that propagates in the z direction in a transparent Kerr medium. $k_0 = k(\omega_0) = n_0\omega_0/c$ and ω_0 are the central wavenumber and frequency of the carrier wave, respectively, $n_0 = 1.5108$ is the linear refractive index of the medium (BK7 glass) and c is the speed of light in vacuum.

The model takes into account diffraction and various nonlinear effects $N(\mathcal{E}^2,\rho)$:

$$N(\mathcal{E}^2,\rho) = i\frac{\omega_0}{c}n_2|\mathcal{E}^2| - \frac{\sigma}{2}(1+i\omega_0\tau_c)\rho - \frac{\beta_K}{2}|\mathcal{E}|^{2K-2}(1-\frac{\rho}{\rho_0}) \tag{2}$$

The first term of equation (2) accounts for the Kerr nonlinearity, with coefficient $n_2 = 2\times10^{-16}$ cm^2/W leading to a critical power for self-focusing $P_{cr} = 3.37$ MW. The second term accounts for plasma absorption and plasma defocusing, $\sigma = 5.23\times10^{-22}$ cm^{-2} is the cross section for inverse Brehmsstrahlung, and $\tau_c = 5.5$ fs is the electron collision time in BK7. Finally the last term in (2) accounts for multiphoton absorption, where $\beta_K = K\hbar\omega_0\rho_0\sigma_K$, and $\sigma_K = 2.4\times10^{-27}$ cm^{16}/W^8/s are the multi-photon ionization coefficients for $K = \left\langle \frac{U_i}{\hbar\omega_0}+1\right\rangle = 3$ photons in BK7 ($U_i = 4$ eV is the bandgap of BK7).

Equation (1) is coupled with an evolution equation for the electron density, which describes plasma generation as a result of the laser pulse interaction with the medium:

$$\frac{\partial\rho}{\partial t} = \sigma_K|\mathcal{E}|^{2K}(\rho_0-\rho) + \frac{\sigma}{U_i}\rho|\mathcal{E}|^2 \tag{3}$$

This equation takes into account multiphoton ionization with rate $W_{\text{MPI}} = \sigma_K I^K$, as well as avalanche ionization where $\rho_0 = 2.1\times10^{28}$ m^{-3} is the number density of neutral atoms.

The refractive index variation is modeled as a lattice of waveguide slabs using the following refractive index distribution:

$$\Delta n(x,y) = \Delta n_0 \sum_{i=1}^{m}\exp\left[-\left(\left(\frac{x}{w_x}\right)^{2K} + \left(\frac{(y-i\Lambda)}{w_y}\right)^{2K}\right)^{1/2}\right] \tag{4}$$

where $\Delta n_0 = -4\times10^{-4}$ is the refractive index modulation, m is the number of waveguides ($m = 7$), $\Lambda = 25$ μm is the lattice period. $w_x = 5$ μm and $w_y = 25$ μm correspond to the waveguide cross section.

[1] P. Panagiotopoulos, N. K. Efremidis, D. G. Papazoglou, A. Couairon, and S. Tzortzakis, Phys. Rev. A **82**, 61803 (2010).

[2] More precisely, a comparison between lattice filament propagation in cylindrically symmetric lattice with dispersion and in (x, y) lattice without dispersion has been numerically performed.