Numerical Model

The numerical model that is used in the simulations resolves a nonlinear Schrödinger type equation:

$$\frac{\partial E}{\partial z} = \frac{i}{2k_o} T^{-1} \Delta_\perp E - i \frac{k^{"}}{2} \frac{\partial^2 E}{\partial t^2} + N\left(E^2, \rho\right) \cdot E + i\, k_o T^{-1} \Delta n(x,y) \cdot E \qquad (1)$$

which describes the evolution of the slowly varying envelope *E(x,y,z,t)* of the electric field $\mathrm{E} = \mathrm{Re}\left[E \exp\left(ik_0 z - i\omega_0 t\right)\right]$ of a laser pulse that propagates in the *z* direction in a transparent Kerr medium. $k_0 = k\left(\omega_0\right) = \frac{n_0 \omega_0}{c}$ and ω_0 are the central wavenumber and frequency of the carrier wave, respectively, n_0 is the linear refractive index of the medium and c is the speed of light in vacuum. The model takes into account diffraction, group velocity dispersion with coefficient $k^{"} = \left.\frac{\partial^2 k}{\partial \omega^2}\right|_{\omega_0}$, and various nonlinear effects $N\left(E^2, \rho\right)$. In particular

$$N\left(E^2, \rho\right) = iT \frac{\omega_0}{c} n_2 \left|E\right|^2 - T^{-1} \frac{\sigma}{2}\left(1 + i\omega_0 \tau_C\right)\rho - \frac{\beta_K}{2} \left|E\right|^{2K-2} \left[1 - \frac{\rho}{\rho_{at}}\right] \qquad (2)$$

where $T = \left(1 + \frac{i}{\omega_0} \frac{\partial}{\partial t}\right)$ is an operator that corresponds to the space-time focusing and self-steepening of the laser pulse. The first term of equation (2) accounts for the Kerr nonlinearity with coefficient n_2 = 3.2x10^{-19} cm^2/W (check value), leading to a critical power for self-focusing Pcr= 3GW. The second term accounts for plasma absorption and plasma defocusing, $\sigma = 5.510^{-20}\, cm^{-2}$ is the cross section for inverse Brehmsstrahlung, and τ_C =350 fs is the collision time in air. Finally the last term in (2) accounts for multiphoton absorption, where $\beta_K = K\hbar\omega_0 \rho_{at} \sigma_K$, and $\sigma_K = 3.4 \times 10^{-96} \mathrm{cm}^{16}/\mathrm{W}^8/\mathrm{s}$ are the multi-photon ionization coefficients for $K = \left\langle \frac{U_i}{\hbar\omega_0} + 1 \right\rangle = 8$ photons in air (U_i is the ionization potential of the medium).

Equation (1) is coupled with an evolution equation for the electron density (3), which describes plasma generation as a result of the laser pulse interaction with the medium:

$$\frac{\partial\rho}{\partial t} = \sigma_K |E|^{2K}(\rho_{at} - \rho) + \frac{\sigma}{U_i}\rho|E|^2 - a\rho^2 \qquad (3)$$

This equation takes into account multiphoton ionization with rate $W_{MPI} = \sigma_K I^K$, as well as avalanche ionization where ρ_{at} is the density of neutral atoms. The last term in (3) accounts for the recombination of free electrons.

Soliton solutions

We look for soliton solutions to a simplified form of Eq. (1), with a lattice consisting of an array of equally spaced negative Δn rods (see inset of Fig. 1(a)). Equation (1) can be written in normalized coordinates $\zeta = z/z_0$, $\xi = x/w_0$, $\eta = y/w_0$, $\psi = E/A$. In particular, we set $z_0 = w_0^2 k_0$, $A^2 = 1/(k_0 n_2 z_0)$ and keep only the diffraction, the Kerr, and the linear potential terms leading to

$$i\psi_\zeta + \frac{1}{2}\left(\psi_{\xi\xi} + \psi_{\eta\eta}\right) + V(\xi,\eta)\psi + |\psi|^2\psi = 0 \qquad (4)$$

In Eq. (4) the potential contrast is related to the index contrast by $V_0 = (\Lambda k_0)^2 \Delta n_0$, where Λ is the lattice period. In our numerical simulations, each rod is modeled by an eighth order supergaussian (mimicking a realistic experimental plasma profile in air), the distance between neighboring waveguides is unity, the diameter of each waveguide is 0.75, whereas $V_0 = 20$.

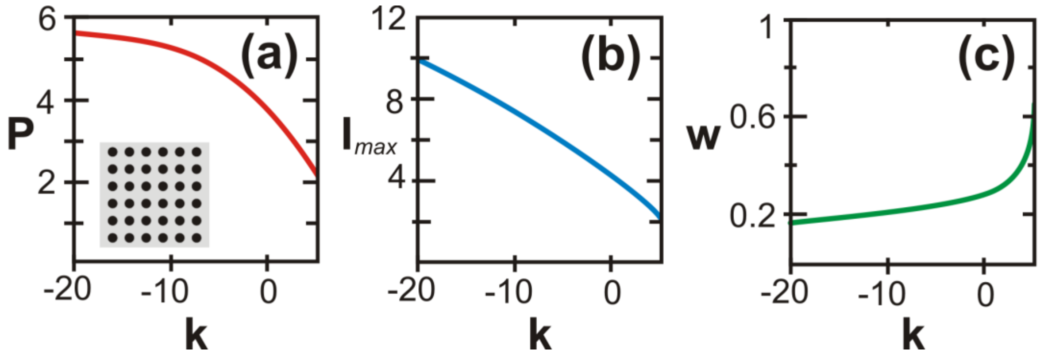

Fig. 1 A family of two-dimensional lattice solitons. (a) Total power P, (inset: graphic representation of the rectangular lattice, black represents lower refractive index), (b) maximum intensity I_{max}, and (c) beam width are shown as a function of the propagation constant k (all values are given in normalized units).

In this type of lattice, light has the tendency to become localized in the high index areas between the low-index leaky-waveguides. Although the potential maxima are not isolated, this lattice supports stable lattice solitons of the form $\psi = u(\xi,\eta)\exp(-ik\zeta)$. We are interested in soliton solutions with eigenvalues in the semi-infinite gap of the spectrum. The properties of this family of stationary solutions are shown in Fig. 1. In particular, the total power $P = \iint |\psi(\xi,\eta)|^2 \, d\xi d\eta$, the maximum intensity I_{max}, and the soliton width w, defined as the averaged value $w = \sqrt{\langle \vec{r}^{\,2} \rangle} = \left(\iint \vec{r}^{\,2} |\psi|^2 d\xi d\eta / P \right)^{1/2}$ are presented as a function of the propagation constant k. The solitons shown in Fig. 1 exhibit both lower and upper power thresholds. The upper power threshold is independent of the lattice type and is equal to the critical power of the NLS equation without a lattice $P_{\max} = P_{cr} \approx \pi \cdot 1.86225 \approx 5.85043$ (norm. units). On the other hand, the lower power threshold P_{min} depends on the parameters of the lattice.