Nonlocal lattice solitons in thermal media

Nikolaos K. Efremidis
Department of Applied Mathematics, University of Crete, 71409 Heraklion, Crete, Greece
(Received 5 March 2008; published 16 June 2008)

We study the properties of solitons in thermal nonlocal media (media having an infinite range of nonlocality) with a periodic refractive index modulation. Such self-localized solutions exist for both signs of the nonlinearity and exhibit a host of features that have no counterpart in local or nonlocal media with finite range nonlocality. Perhaps the most intriguing property is that families of bright solitary pulses can exist inside the bands of the band structure. This property is attributed to the infinite range of nonlocality as well as to the boundary conditions applied to the system. Families of solitons bifurcating from higher-order band edges, otherwise unstable in local media, are found to be stable.

DOI: 10.1103/PhysRevA.77.063824 PACS number(s): 42.65.Tg, 42.65.Jx

I. INTRODUCTION

The study of nonlinear wave propagation in optical systems with a periodic index modulation has attracted a lot of attention during the last years [1]. In such systems, self-localized beams or discrete solitons can exist due to the interplay between the effective lattice diffraction and nonlinearity. Solitons in periodic waveguide structures have been predicted in [2] and were experimentally observed 10 years later [3]. In two-dimensional settings lattice solitons have been observed in [4] using an optical induction technique [5,6]. In most of the studies in optical periodic lattices, local nonlinear behavior has been considered [7–10].

Nonlocal nonlinear behavior in a system is related to a nonlinear refractive index change that depends not only on the local beam intensity but also on the intensity of the surrounding region according to a specific distribution. As a result, an optical beam propagating in a nonlocal medium has a higher tendency to become less localized. In physical settings, nonlocality can be associated with some sort of transport mechanism. Nonlocal nonlinearities can occur in thermal nonlinear media, where nonlocality is an outcome of heat diffusion due to light absorption [11–14]. In liquid crystals [15–19] the nonlinearity is a result of long-range molecular reorientations. Nonlocal molecular interactions are also possible in Bose-Einstein condensates [20,21].

Due to the nature of the nonlinearity, solitons in such systems exhibit several distinct properties that are not possible in local settings. For example, long-range nonlinearities can eliminate wave collapse and stabilize multidimensional solitons [22–24]. In addition, interacting π-out-of-phase solitons can attract for strong enough nonlocality or can result to the formation of soliton bound states [25–28]. Several families of otherwise unstable solitons, such as ring vortices, are known to be stabilized in nonlocal media [14–30]. In the particular setting of thermal nonlocal media, the existence of families of solitons as well as their properties highly depends on the boundary conditions imposed [31].

Discrete solitons have been studied in media with finite range nonlocality. In particular, discrete propagation in an array of channel waveguides in undoped nematic liquid crystals has been experimentally and theoretically considered in [32] (see also [33] for a recent review). In [34], it was shown that the impact of nonlocality in media with imprinted lattice results in a significant reduction of the Pierls-Nabaro potential. In [35] breatherlike collision of gap solitons has been studied.

In this work, the properties of nonlocal solitons in media with a periodic refractive index modulation are examined. The case of thermal nonlinearity, where the intensity of the light beam under steady-state conditions satisfies the heat diffusion equation, is investigated. Both self-focusing and self-defocusing configurations are analyzed. It is shown that self-localized soliton solutions exhibit existence and stability properties fundamentally different from those encountered in either local media or nonlocal media with finite range nonlocality. Perhaps the most intriguing feature is the existence of families of solutions inside the linear spectrum (i.e., the bands of the band structure). This property is attributed to (i) the infinite range of nonlocality and (ii) the presence of boundaries. It should be pointed out that such solitons are fundamentally different from the codimension-1 solutions known as embedded solitons [36,37]. In particular, thermal nonlocal lattice solitons can exist anywhere inside a specific range of values of the propagation constant [that can contain band(s) and/or a part of a band]. Two families of solutions bifurcate from each band edge. Their stability is examined by direct numerical simulation. It is found that at least one of these two families of solutions originating from all possible band edges and for both signs of the nonlinearity is always stable whereas the second can be stable or unstable. Notice that in Kerr media such higher-order gap solitons are unstable.

II. MODEL

Let us start by considering paraxial propagation in a one-dimensional optical system with nonlocal nonlinearity in the presence of a periodic potential as given by

$$iu_z + \frac{1}{2}u_{xx} - V(x)u + \gamma uv = 0, \tag{1}$$

$$\sigma v_{xx} - (1-\sigma)v = -|u|^2. \tag{2}$$

In the above normalized set of equations, we assume asample of length L, $-L/2 \le x \le L/2$, $V(x)$ is a periodic potential, $0 \leq \sigma \leq 1$ determines the range of nonlocality, and $\gamma = \pm 1$ for self-focusing or self-defocusing nonlinearity. Here, we consider sinusoidal potentials that have refractive index maximum or minimum at the center of the medium, i.e., $V(x) = V_1(x) = V_0 \sin^2(\pi x / x_0)$ or $V(x) = V_2(x) = V_0 \cos^2(\pi x / x_0)$, respectively. For simplicity, we restrict ourselves to potentials of amplitude $V_0 = 10$ with period $x_0 = 2$. According to the values of σ one can distinguish three different cases. (i) $\sigma = 0$: In this limit, Eqs. (1) and (2) exhibit local behavior and are identical to the nonlinear Schrödinger equation with a Kerr (cubic) nonlinearity. (ii) $0 < \sigma < 1$: The behavior is nonlocal but the range of nonlocality is finite. As σ increases the range of nonlocality also increases. Such equations can describe propagation in nematic liquid crystals. The nonlocal field $v(x)$ represents the reorientation angle of the molecules [19]. (iii) $\sigma = 1$: The range of nonlocality is infinite. In samples of finite width and due to boundary conditions, the range of nonlocality becomes comparable to the width of the sample. In physical settings, the nonlocality has an infinite range in media with thermal nonlinearities [14]. Thus $v(x)$ satisfies the heat diffusion equation (2) in steady-state conditions. In this paper, our attention is concentrated on this latter limit, i.e., $\sigma = 1$.

Notice that Eq. (2) can be solved for $v(x)$ using Green's functions, i.e.,

$$v(x) = \frac{v_0 \cosh(\gamma x)}{\cosh(\gamma L/2)} + \int_{-L/2}^{L/2} G(x, x') |u(x')|^2 dx', \tag{3}$$

where

$$G(x_1, x_2) = \frac{\prod_{j=1,2} \sinh\{\gamma[L/2 - \kappa(x_j, x_{3-j}) x_j]\}}{\gamma \sigma \sinh(\gamma L)}, \tag{4}$$

$\kappa(x, x') = \mathrm{sgn}(x - x')$ and $\gamma = \sqrt{(1-\sigma)/\sigma}$. In the above expressions, constant amplitude boundary conditions $v(\pm L/2) = v_0$ are considered. In thermal media ($\sigma = 1$), such boundary conditions account for constant temperature at the boundaries, something that is possible if the boundaries of the sample are connected to a heat sink. When $\sigma = 1$ using the transformation $v \to v + v_0$ the aforementioned boundary conditions are simplified to $v(\pm L/2) = 0$. When $\sigma = 1$ and $v(\pm L/2) = 0$ the Green's function takes the form

$$G(x, x') = \frac{1}{L}\left(\frac{L}{2} + \kappa(x, x') x'\right)\left(\frac{L}{2} - \kappa(x, x') x\right). \tag{5}$$

Finally, in the local limit ($\sigma = 0$) it is $G(x, x') = \delta(x - x')$. Equations (1) and (2) have two integrals of motion: The total power

$$P = \int_{-L/2}^{L/2} |u|^2 dx \tag{6}$$

and the Hamiltonian

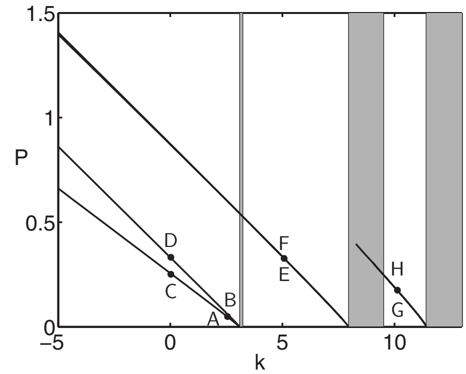

FIG. 1. Total power P as a function of the propagation constant k of various families of self-focusing ($\gamma = 1$) nonlocal lattice solitons localized at the center of the medium. The shaded gray and white areas represent bands and band gaps, respectively. The white areas from left to right are the semi-infinite band gap and the first two finite gaps. Notice that the difference in the total power of the modes bifurcating from the second and the third bands is very small and thus the curves almost coincide.

$$H = \frac{1}{2}\int_{-L/2}^{L/2} \left(\left| \frac{\partial u}{\partial x} \right|^2 - 2|u(x)|^2 V'(x) - \gamma |u(x)|^2 \int_{-L/2}^{L/2} G(x, x') |u(x')|^2 dx' \right) dx, \tag{7}$$

where $V'(x) = V(x) + v_0 \cosh(\gamma x)/\cosh(\gamma L/2)$. Making use of the functional derivative of the Hamiltonian, Eqs. (1) and (2) can be expressed as

$$i \frac{\partial u}{\partial z} = \frac{\delta H}{\delta u^*}. \tag{8}$$

When the input beam propagates linearly inside the nonlocal medium, Eqs. (1) and (2) reduce to

$$i u_z + \frac{1}{2} u_{xx} - V(x) u = 0. \tag{9}$$

The properties of Eq. (9), which are independent of σ, can be analyzed using Floquet-Bloch theory [38]. The linear band structure of the system can be found, for example, in Refs. [7,8]. In Fig. 1 the shaded and white areas represent the bands and band gaps of the band structure, as a function of the propagation constant k. Notice that the jth band can be defined in the domain $k_{L,j} \leq k \leq k_{R,j}$ ($k_{L,j}$ and $k_{R,j}$ being the left- and right-hand boundaries of the jth band of the band structure).

III. SOLITON SOLUTIONS

In the nonlinear case, Eqs. (1) and (2) support families of lattice solitons. We analyze the properties of such solutions for media with an infinite range nonlocality ($\sigma = 1$). We assume that Eqs. (1) and (2) support stationary solutions of the form $u(x) = \psi(x) \exp(-ikz)$, where $\psi(x)$ and k are real, and thus

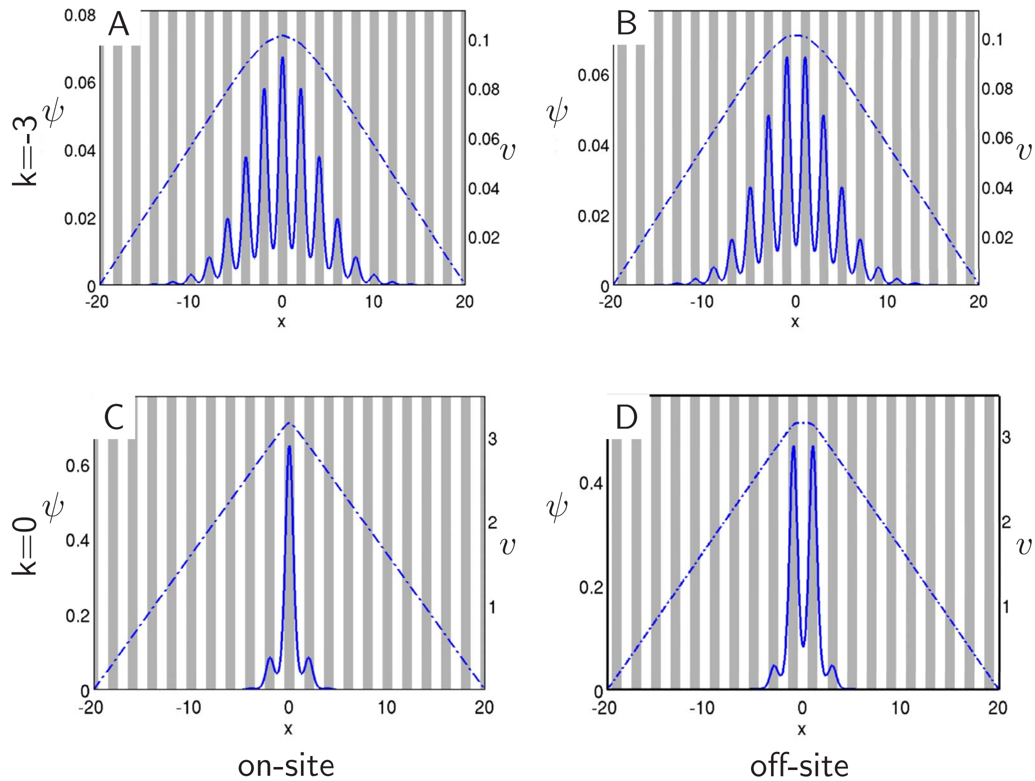

FIG. 2. (Color online) Amplitudes of the optical wave ψ (solid line) and the nonlocal field v (dashed-dotted line). These solutions belong to self-focusing soliton families that bifurcate from the base of the first band ($k_{L,1}$). The shaded gray areas represent high refractive index. On the left-hand (right-hand) column modes centered at a refractive index maximum (minimum) are depicted. The P-k characteristics of solitons (A)–(D) are shown in Fig. 1.

$$k\psi + \frac{1}{2}\psi_{xx} - V(x)\psi + \gamma\psi v = 0, \tag{10}$$

$$v_{xx} = -\psi^2. \tag{11}$$

Localized solutions of Eqs. (10) and (11) are found numerically, using Newton's iteration method. We consider two separate cases according to the sign of the nonlinearity which can be self-focusing ($\gamma=1$) or self-defocusing ($\gamma=-1$). In the self-focusing case, the numerically found existence curves (power vs propagation constant) are presented in Fig. 1. The bifurcation points are located on the left-hand boundary of each band. From each bifurcation point two different families of solutions appear. One family is centered at a refractive index maximum $[V(x)=V_1(x)]$ and one at a refractive index minimum $[V(x)=V_2(x)]$. The first three pairs of families of localized solutions, which bifurcate from the edges of the first three bands ($k_{L,j}, j=1,2,3$), are depicted in Fig. 1.

Two families of solutions bifurcate from the base of the first band (inside the first Brillouin zone) $k_{L,1}$ into the semi-infinite band gap. Typical profiles of the field amplitude and of the corresponding nonlocal field are shown in Fig. 2. Notice that as the total power increases, solitons belonging to the same family have the tendency to become more spatially localized inside the lattice. When the eigenvalue is close to the band edge of the bifurcation point, the form of the soliton resembles the corresponding Floquet-Bloch mode which is modulated by an envelope. In discrete periodic systems the modes shown on the left-hand (right-hand) side of Fig. 2 are known as "on-site" ("off-site"). Here, we will extend this definition by denoting as "on-site" and "off-site" all modes that are centered at a refractive index maximum or minimum, respectively. As can be seen in Fig. 1, the power of the "off-site" mode is substantially higher than the power of the "on-site" mode for the same propagation constant k. Thus the "on-site" mode is energetically favorable. The stability of the solitons is tested by direct simulation. The initial condition applied is the numerically found lattice soliton solution with a random perturbation that has maximum amplitude equal to 0.1% of the maximum soliton amplitude. In the simulations the "on-site" family is found to be stable, whereas the "off-site" is unstable. The instability does not lead to radiation. It is observed that the "off-site" mode exhibits oscillatory behavior as it asymptotically propagates toward the corresponding "on-site" soliton with the exact same power and a smaller propagation constant k (Fig. 3). The same scenario is observed in all the cases where instability occurs: The unstable mode switches to the corresponding stable mode that originates from the same bifurcation point.

The other two pairs of families of self-focusing lattice solitons bifurcate from the edge of the second band ($k_{L,2}$) and the base of the third band ($k_{L,3}$). In Fig. 4 typical amplitude profiles are shown. It should be pointed out that such solitons exhibit fundamentally different existence and stability properties as compared to lattice solitons in media with either local or finite-range nonlocal nonlinearity. In particular, for $\sigma \neq 1$ such solutions are known as "gap solitons" because

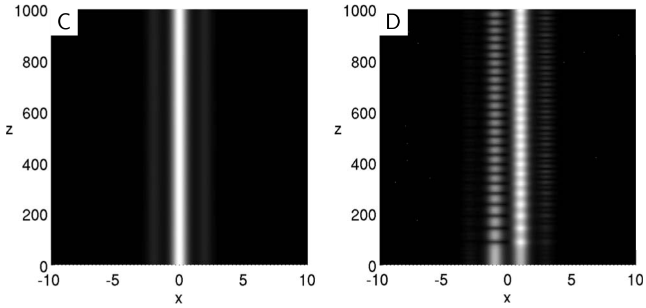

FIG. 3. Propagation dynamics of the nonlocal lattice solitons shown on the bottom of Fig. 2 [(C), (D)] for $k=0$). Left-hand side: Stable propagation of the "on-site" soliton. Right-hand side: Unstable dynamics of the "off-site" mode.

their propagation constant always resides within a finite band gap between two bands [7,8,35]. However, our results reveal that, if $\sigma=1$, this property no longer holds true (and thus the term gap soliton is not valid). To elucidate this, let us first examine the two families of solutions that bifurcate from the edge of the second band (for $k=k_{L,2}$) into the first finite gap. In Fig. 1 one can see that as the total power increases, the existence curve penetrates inside the first band, and even further into the semi-infinite band gap, i.e., solutions exist for values of the propagation constants $-\infty<k<k_{L,2}$. To our knowledge these families of nonlocal lattice solitons are the first one-dimensional solutions that can exist anywhere inside a band (or dispersion-diffraction curve). In contrast, localized solutions known as embedded solitons [36,37] can exist inside the linear spectrum for discrete values of the propagation constant (i.e., they are isolated solutions or have codimension-1). Embedded solitons are possible if the spectrum of the linearized system has (at least) two branches, one corresponding to exponentially localized solutions and the other to radiation modes.

Physically, it is possible to explain nonlinear localization inside the bands in terms of the total (linear and nonlinear) refractive index change induced by the nonlocal medium $F(x)=-V(x)+\gamma v(x)$. To derive asymptotic results, we consider the simplified case where most of the soliton intensity is concentrated at $x=a$, i.e., $|u(x)|^2\simeq u_0^2\delta(x-a)$ resulting in $v(x)=u_0^2G(x,a)$. For $\sigma\neq 1$, $v(x)$ is a function that exponential goes to zero at the edge of the sample ($x\to\pm L/2$). This means that nonlinearity modifies the refractive index in a localized area that surrounds the high intensity region, but not everywhere. As a result, the soliton tails see an environment identical to the linear, i.e., solitons can be found only in the band gaps of the linearized system (where a stable and an unstable manifold exist). On the contrary, when $\sigma=1$ the nonlinear refractive index is nonzero everywhere along the medium. In particular, $v(x)$ is a function which has a maximum at $x=a$ and linearly goes to zero at $x=\pm L/2$. This linear relation of the nonlocal field is verified numerically (see Figs. 2, 4, and 6). Thus, the refractive index is modified everywhere along the medium and the soliton tails see a

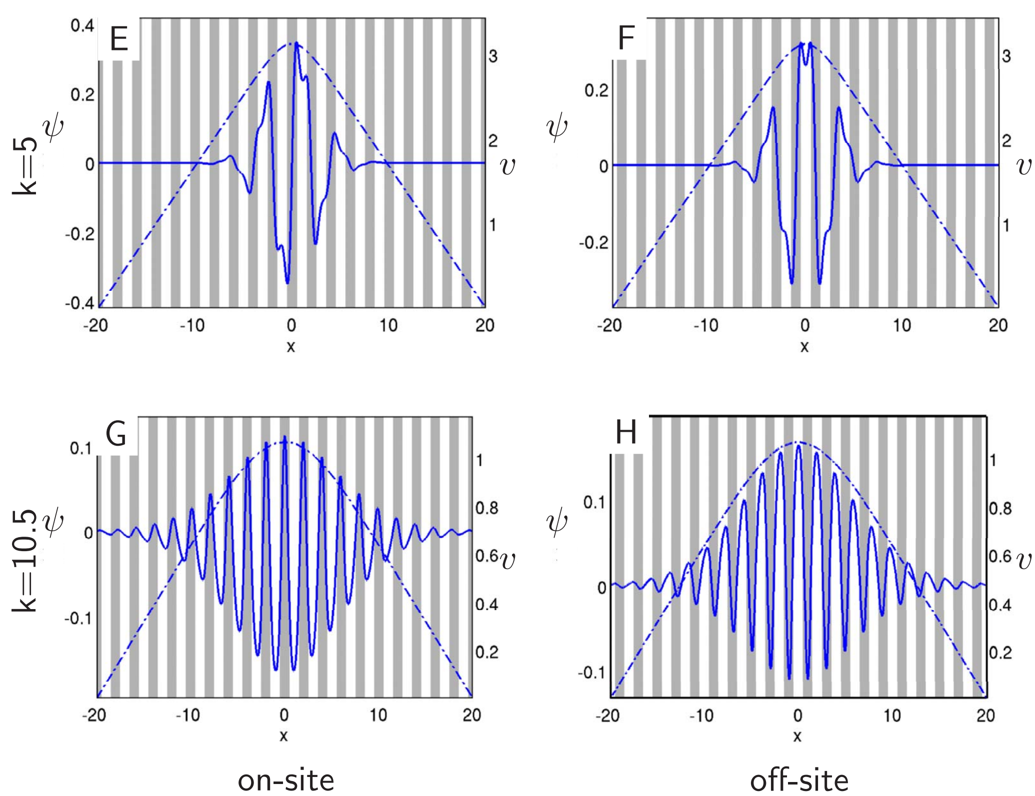

FIG. 4. (Color online) Amplitudes of the optical wave ψ (solid line) and the nonlocal field v (dashed-dotted line). Top and bottom panels show solutions that belong to self-focusing soliton families which bifurcate from the edge of the second band ($k_{L,2}$) and the base of the third band ($k_{L,3}$), respectively. The shaded gray areas represent high refractive index. On the left-hand (right-hand) column modes centered at a refractive index maximum (minimum) are shown. The P-k characteristics of solitons (E)–(H) are shown in Fig. 1.

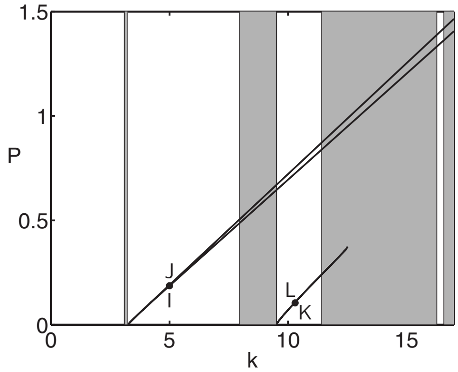

FIG. 5. Same as in Fig. 1 for self-defocusing nonlinearity ($\gamma=-1$).

modified index environment. The conditions for soliton existence rely on this modified potential $F(x)$.

Considering the stability of such nonlocal lattice solitons, our numerical simulations showed that the on-site mode (which has the lowest power for the same propagation constant) is stable for all values of k. On the other hand, the off-site mode can be either stable or unstable according to the propagation constant. In particular, for small values of the total power no instability is observed, whereas for larger values of P solitons turn unstable. For example, when $k=5$ no instability is observed, whereas for $k=0$ the soliton is unstable. The instability dynamics leads to similar behavior as in the case of off-site solitons bifurcating from $k_{L,1}$: The unstable mode slowly evolves toward the corresponding stable mode that originates from the same bifurcation point and has the same total power and a slightly different propagation constant.

Finally, two self-focusing families of solutions (Fig. 1) bifurcate from the base of the third band ($k_{L,3}$). Typical field profiles of such solitons are depicted on the bottom panel of Fig. 4. Notice that the P-k curves penetrate inside the second band, but only for a finite depth. After a critical value of the propagation constant k_1 ($k_{L,2}<k_1<k_{R,2}$), lattice solitons cease to exist due to the interaction with the linear spectrum, i.e., the possible values of k lie in the range $k_1<k<k_{L,3}$. Notice that the difference in the total power between the on-site and off-site modes is very small to be traceable in Fig. 1. In our simulations both modes are found to be stable for all values of k. The physical explanation for the spatial "destabilization" of this family of solutions at k_1 is rendered to the number of modes supported by the total lattice potential $F(x)$. Thus, the higher-order nonlinear mode that this family of solitons represents has a cutoff at k_1.

In Fig. 5 the existence P-k curves of self-defocusing ($\gamma=-1$) nonlocal lattice solitons are depicted. Soliton solutions bifurcate from the right-hand boundaries of the bands ($k_{R,j}$). Again, on-site and off-site families of solutions appear from each bifurcation point. In contrast to the self-focusing case, soliton solutions do not exist in the semi-infinite band gap of the spectrum. In Fig. 6 typical profiles of the optical

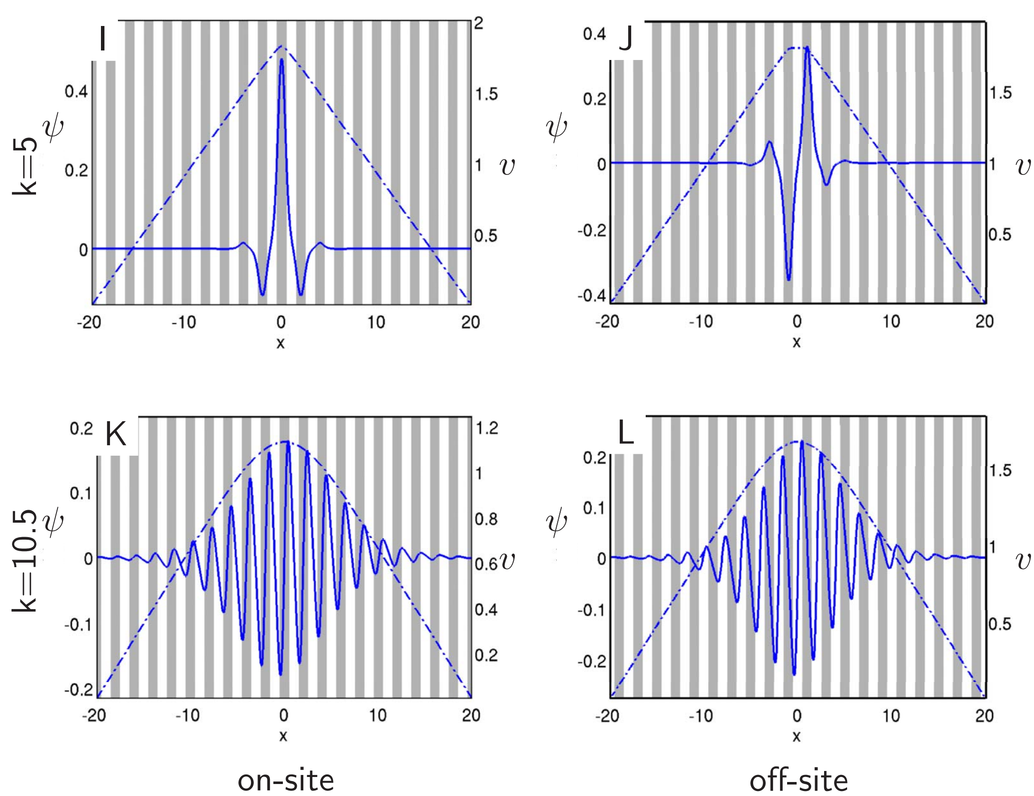

FIG. 6. (Color online) Amplitudes of the optical wave ψ (solid line) and the nonlocal field v (dashed-dotted line). Top and bottom panels depict solutions that belong to self-defocusing soliton families which bifurcate from the edge of the first band ($k_{R,1}$) and the base of the second band ($k_{R,3}$), respectively. The shaded gray areas represent high refractive index. On the left-hand (right-hand) column modes centered at a refractive index maximum (minimum) are shown. The P-k characteristics of solitons (I)–(L) are shown in Fig. 5.

and nonlocal field of such lattice solitons are shown for the families that bifurcate from the first two bands. Two modes bifurcate from the edge of the first band $k_{R,1}$ into the first finite band gap. As the total power increases, the propagation constant of these solitons subsequently penetrates through higher bands and band gaps, i.e., such solutions exist for $k_{R,1} < k < \infty$. The on-site mode is found to be stable. The power difference between the two families of solutions for the same value of the propagation constant is significant. Thus, the off-site solution (which has the highest P for the same propagation constant) is found to be numerically unstable. Finally, two families of self-defocusing lattice solitons bifurcate from the base of the second band $k_{R,2}$ into the second band gap. The propagation constant of these solutions, penetrates further into the third band. Both of these families are found to be stable.

IV. CONCLUSIONS

In conclusion, we have studied the properties of solitons in thermal nonlocal media (media having an infinite range of nonlocality) with a periodic refractive index modulation. Self-localized solutions were found to exhibit a host of features that have no counterpart in local or finite-range nonlocal media. Perhaps the most unexpected property is that specific families of bright solitary pulses can exist inside the bands of the band structure. This property is attributed to the infinite range of nonlocality as well as to the boundary condition applied to the system. Families of lattice solitons, otherwise unstable in local media, were found to be robust.

ACKNOWLEDGMENT

The author would like to thank Professor K. Hizanidis for helpful discussions and comments concerning this work.

[1] D. N. Christodoulides, F. Lederer, and Y. Silberberg, Nature (London) **424**, 817 (2003).
[2] D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 794 (1988).
[3] H. S. Eisenberg, Y. Silberberg, R. Morandotti, A. R. Boyd, and J. S. Aitchison, Phys. Rev. Lett. **81**, 3383 (1998).
[4] J. W. Fleischer, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Nature (London) **422**, 147 (2003).
[5] N. K. Efremidis, S. Sears, D. N. Christodoulides, J. W. Fleischer, and M. Segev, Phys. Rev. E **66**, 046602 (2002).
[6] J. W. Fleischer, T. Carmon, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Phys. Rev. Lett. **90**, 023902 (2003).
[7] P. J. Y. Louis, E. A. Ostrovskaya, C. M. Savage, and Y. S. Kivshar, Phys. Rev. A **67**, 013602 (2003).
[8] N. K. Efremidis and D. N. Christodoulides, Phys. Rev. A **67**, 063608 (2003).
[9] D. Neshev, E. Ostrovskaya, Y. Kivshar, and W. Krolikowski, Opt. Lett. **28**, 710 (2003).
[10] H. Martin, E. D. Eugenieva, Z. Chen, and D. N. Christodoulides, Phys. Rev. Lett. **92**, 123902 (2004).
[11] A. G. Litvak, JETP Lett. **4**, 230 (1966).
[12] F. W. Dabby and J. R. Whinnery, Appl. Phys. Lett. **13**, 284 (1968).
[13] A. G. Litvak, V. A. Mironov, G. M. Fraiman, and A. D. Yunakovskii, Sov. J. Plasma Phys. **1**, 31 (1975).
[14] C. Rotschild, O. Cohen, O. Manela, M. Segev, and T. Carmon, Phys. Rev. Lett. **95**, 213904 (2005).
[15] E. Braun, L. P. Faucheux, and A. Libchaber, Phys. Rev. A **48**, 611 (1993).
[16] M. Warenghem, J. Henninot, and G. Abbate, Opt. Express **2**, 483 (1998).
[17] M. A. Karpierz, M. Sierakowski, M. Świllo, and T. Woliński, Mol. Cryst. Liq. Cryst. Sci Techol., Sect. A **320**, 157 (1998).
[18] M. Peccianti, A. D. Rossi, G. Assanto, A. De Luca, C. Umeton, and I. C. Khoo, Appl. Phys. Lett. **77**, 7 (2000).
[19] C. Conti, M. Peccianti, and G. Assanto, Phys. Rev. Lett. **91**, 073901 (2003).
[20] A. Parola, L. Salasnich, and L. Reatto, Phys. Rev. A **57**, R3180 (1998).
[21] A. Griesmaier, J. Werner, S. Hensler, J. Stuhler, and T. Pfau, Phys. Rev. Lett. **94**, 160401 (2005).
[22] S. Turitsyn, Theor. Math. Phys. **64**, 797 (1985).
[23] V. M. Pérez-García, V. V. Konotop, and J. J. García-Ripoll, Phys. Rev. E **62**, 4300 (2000).
[24] O. Bang, W. Krolikowski, J. Wyller, and J. J. Rasmussen, Phys. Rev. E **66**, 046619 (2002).
[25] D. W. McLaughlin, D. J. Muraki, and M. J. Shelley, Physica D **97**, 471 (1996).
[26] M. Peccianti, K. A. Brzdakiewicz, and G. Assanto, Opt. Lett. **27**, 1460 (2002).
[27] P. D. Rasmussen, O. Bang, and W. Królikowski, Phys. Rev. E **72**, 066611 (2005).
[28] W. Hu, T. Zhang, Q. Guo, L. Xuan, and S. Lan, Appl. Phys. Lett. **89**, 071111 (2006).
[29] D. Briedis, D. Petersen, D. Edmundson, W. Krolikowski, and O. Bang, Opt. Express **13**, 435 (2005).
[30] A. I. Yakimenko, Y. A. Zaliznyak, and Y. Kivshar, Phys. Rev. E **71**, 065603(R) (2005).
[31] B. Alfassi, C. Rotschild, O. Manela, M. Segev, and D. N. Christodoulides, Opt. Lett. **32**, 154 (2007).
[32] A. Fratalocchi, G. Assanto, K. A. Brzdakiewicz, and M. A. Karpierz, Opt. Lett. **29**, 1530 (2004).
[33] G. Assanto, A. Fratalocchi, and M. Peccianti, Opt. Express **15**, 5248 (2007).
[34] Z. Xu, Y. V. Kartashov, and L. Torner, Phys. Rev. Lett. **95**, 113901 (2005).
[35] Y. Lin, I.-H. Chen, and R.-K. Lee, J. Opt. A, Pure Appl. Opt. **10**, 044017 (2008).
[36] A. R. Champneys, B. A. Malomed, and M. J. Friedman, Phys. Rev. Lett. **80**, 4169 (1998).
[37] J. Yang, B. A. Malomed, and D. J. Kaup, Phys. Rev. Lett. **83**, 1958 (1999).
[38] C. Kittel, *Introduction to Solid State Physics* (Wiley, New York, 1986).