Surface optical Bloch oscillations in semi-infinite waveguide arrays

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We predict that surface optical Bloch oscillations can exist in semi-infinite waveguide arrays with a linear index variation, if the array parameters close to the boundary are appropriately perturbed. The perturbation is such that the surface states obtain the Wannier–Stark ladder eigenvalues of the unperturbed infinite array. The number of waveguides, whose parameters need to be controlled, decreases with increasing ratio of index gradient over coupling. The configuration can find applications as a "matched" termination of waveguide arrays to eliminate the distortion of Bloch oscillations due to reflection on the boundaries.
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Optical Bloch oscillations (BO) in waveguide arrays (WGAs) [1] is the optical analogue of the celebrated quantum-mechanical phenomenon addressed 80 years ago by Zener [2] and Bloch [3]: when a dc electric field is applied to a crystal, free electrons undergo a periodic oscillatory motion, as a result of repeated phases of acceleration, Bragg-reflection and deceleration. The phenomenon was first observed in the 1990s, in artificial semiconductor superlattices, through time-resolved four-wave mixing experiments [4], and by the coherent electromagnetic radiation emitted by the oscillating electrons [5]. The oscillations can alternatively be explained by the existence of localized eigenfunctions with identical profiles called Wannier–Stark (WS) states, which have eigenvalues that are equidistant, and thus form the so-called WS ladder [6]. From this viewpoint, BO are the result of the periodic beating of the excited WS states.

As was theoretically predicted [1] and experimentally verified [7,8], a completely analogous phenomenon to electron BO emerges in optical WGAs, with the periodic lattice acting as the crystal and a linear variation of the waveguides' effective index acting as the electric potential. The index ramp can be imposed electro-optically [1], thermo-optically [7], by varying the waveguide width [8], or by imparting curvature to the waveguides [9]. In such a configuration, an input light beam propagates without diffracting along a sinusoidal path, as a result of repeated total internal reflections (TIRs) and Bragg reflections from the low- and high-index sides of the array, respectively. Using coupled-mode theory, this system was analytically shown to possess localized eigenmodes, whose amplitude profiles are shifted copies of the function $J_{-n}(2\kappa/a)$ (κ being the coupling coefficient and a the propagation constant gradient) and whose eigenvalues are equally separated by a [1]. The beating of these optical WS states leads to *optical* (or photonic) BO and perfect periodic revivals of the input condition at intervals $2\pi/\alpha$. BO and WS states have also been predicted and observed in other optical systems, such as dielectric superlattices [10], chirped Moiré gratings [11], 2D photonic lattices [12] and plasmonic WGAs [13].

One of the most interesting features of BO in WGAs is the periodic recurrence of the input excitation, which implies the ability to transmit arbitrary discrete beams without diffraction over array lengths equal to integer multiples of the BO period. Perfect revivals with arbitrary periods can also occur in WGAs after careful engineering of their parameters [14]. However, there is still an important point to be considered, namely the fact that a perfect ladder of states, which is required for perfect BO, implies an infinite WGA. In a finite WGA, only the states sufficiently confined to its interior approach ideal WS states and thus conform to the ladder rule. On the contrary, at the high- and low-index edges, *surface states* (SSs) are supported, which have eigenvalues that deviate positively and negatively from the ladder, respectively, thus forbidding perfect revivals for input beams that happen to excite them. For example, in Fig. 1(a), a Gaussian beam undergoes perfect BO (and revivals) being sufficiently away from the boundary of a WGA. By shifting the input to the right, as in Fig. 1(b), so that Bragg reflection is inhibited by reflection from the boundary, the revivals are completely destroyed, even though the beam continues to propagate in an oscillatory (breathing) mode, still being confined by TIR on the left. Similar distortion occurs when the sign of the index gradient is reversed.

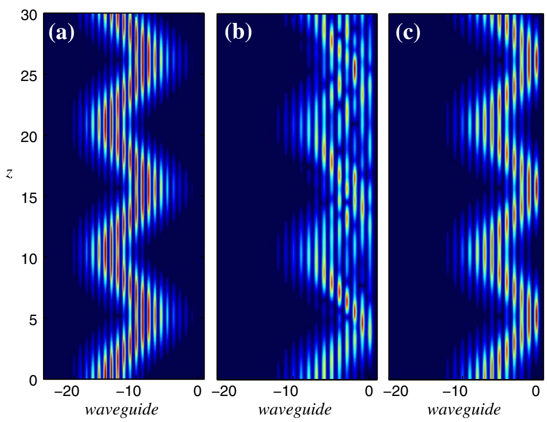

Fig. 1. (Color online) (a) BO of a discrete Gaussian beam in the interior of a WGA with 30 elements. (b) Distortion due to excitation of SS. (c) SBO after engineering the array parameters ($a = 0.6$, $\kappa = 1$). Shown here (and in Figs. 3 and 5) is the field amplitude as obtained by numerically solving system of Eq. (1) with a fourth order Runge–Kutta scheme.

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Thus, the question is whether undistorted BO can occur near the boundary of a WGA. In this Letter, we show that the answer is affirmative. In particular, we predict that optical BO, here termed *surface* optical *BO* (SBO), can occur in semi-infinite WGAs with a linear index variation, provided that the array parameters close to the boundary are appropriately perturbed. The perturbation is such that the truncated WGA supports a perfect, *semi-infinite ladder* of SSs whose beating sustains SBO and periodic revivals. Moving away from the boundary, the perturbation vanishes asymptotically and the SSs evolve to ideal WS states of an infinite array. In practice, the number of elements, that actually need to be engineered, decreases with increasing index-gradient–over-coupling ratio, due to the stronger state localization achieved with larger a/κ. To determine the perturbation parameters, we employ the numerical scheme described below.

Under the validity of coupled-mode theory and nearest-neighbor interactions, the evolution of the power-normalized mode amplitudes in a semi-infinite WGA is expressed through the equations

$$(id/dz + a_n)\psi_n + \kappa_{n-1}\psi_{n-1} + \kappa_n\psi_{n+1} = 0, \qquad (1)$$

where integer $n \le 0$ denotes the waveguide, z is the propagation distance, κ_n is the field coupling coefficient between waveguides n and $n+1$, and a_n is the propagation constant detuning of waveguide n from an average β_0. At the boundary ($n = 0$), the last term in Eq. (1) must be omitted. In an infinite array, $a_n = na$ and coupling is uniform $\kappa_n = \kappa$, resulting in an infinite ladder of WS states with eigenvalues $\lambda_m = ma$, m indicating the waveguide around which WS state m is confined. In the semi-infinite case, however, a_n, κ_n have to be engineered so that a semi-infinite set of SSs are supported with eigenvalues $\lambda_m = ma$, $m \le N$. Obviously, as we move away from the boundary ($n \to -\infty$), the limits $a_n \to na$, $\kappa_n \to \kappa$ ensure that the SSs evolve into ideal WS states.

Unlike the infinite WGA case [1], the problem addressed here has no known analytical solution, hence one must proceed numerically. Since a_n, κ_n have known limits, we assume that the engineering of parameters must take place over a sufficient, but finite, number of N waveguides. We thus assume $2N$ parameters to be controlled, i.e. detunings $a_{-N+1}, \ldots, a_0$ and couplings $\kappa_{-N}, \ldots, \kappa_{-1}$, while the rest are left unperturbed, i.e. $a_n = na$, for $n \le -N$, and $\kappa_n = \kappa$, for $n \le -N-1$. Subsequently, for an SS with eigenvalue $\lambda_m = ma$, the evolution Eq. (1) becomes

$$(a_n - \lambda_m)\psi_{n,m} + \kappa_{n-1}\psi_{n-1,m} + \kappa_n\psi_{n+1,m} = 0, \qquad (2)$$

where $\psi_{n,m}$ is the amplitude of SS m in waveguide n. Now, by assuming that the amplitudes of waveguides $n \le -N$ follow the WS profile $\psi_{n,m} = J_{m-n}(2\kappa/a)$, Eq. (2) is automatically satisfied in waveguides $n \le -N-1$

The remaining $N+1$ equations are to be satisfied by optimizing the parameters. Assuming a number of M *test* SSs, one obtains a set of $M(N+1)$ nonlinear equations for $2N + MN$ unknown variables, namely the $2N$ parameters a_n, κ_n and the MN amplitudes $\psi_{n,m}$. Obviously, for $M = 2N$, the number of equations equals that of the unknowns. The set of nonlinear equations is solved through a Newton–Raphson method with the initial guess being the unperturbed parameters, i.e. $a_n^0 = na$, $\kappa_n^0 = \kappa$, and $\psi_{n,m}^0$ is an SS of an unperturbed finite array that is sufficiently long to mimic the semi-infinite one. Concerning the λ_m values of the test SSs, an obvious choice is the ladder, i.e. $\lambda_m/a = \{-M+1, \ldots, -1, 0\}$. Other choices are also eligible, as long as the eigenvalues differ by integer multiples of a. The ladder choice, however, leads to less varying values for a_n, κ_n and, hence, is the most practical one.

Figure 2(a) shows the parameters obtained for a WGA with $a = 0.6$, $\kappa = 1$. A number of $N = 12$ elements was sufficient for the parameters to converge. As it might be intuitively expected, the detunings close to the edge must be lowered to restore the positive deviation of the SSs' eigenvalues from the ladder, while the couplings must also be apodized to facilitate a smooth array termination. Note also how the parameters tend asymptotically to the expected limits as $n \to -N$. In Figs. 2(b)–2(e) the mode amplitudes $\psi_{n,m}$ of the first four of the M SSs are shown, as obtained by the solution of the nonlinear system, with eigenvalues 0, $-a$, $-2a$, $-3a$. For comparison, mode amplitudes of the corresponding SSs of the unperturbed WGA are superposed, with their deviating eigenvalues $1.6a$, $-0.25a$, $-1.7a$, $-2.9a$. With decreasing $\lambda = -3a, -4a, \ldots$, the SSs move to the left and approach the WS profile. Already from SSs $\psi_{n,-3}$ [Fig. 2(e)], the WS profile starts manifesting itself, as indicated by the left (TIR-confined) train of in-phase amplitudes and the right (Bragg-confined) train of anti-phase amplitudes.

Having engineered the SS eigenvalues, the semi-infinite WGA is capable of undistorted SBO. This is verified by Fig. 1(c), where the input beam of Fig. 1(b) is launched into a WGA with the parameters of Fig. 2(a). Now the beam interacts with the boundary without losing periodicity along z, reconstructing itself perfectly at intervals $2\pi/a$, the SBO period. It is also interesting to see how this WGA responds to single-waveguide excitation (Green's function) [Figs. 3(b) and 3(c)]. Notice the

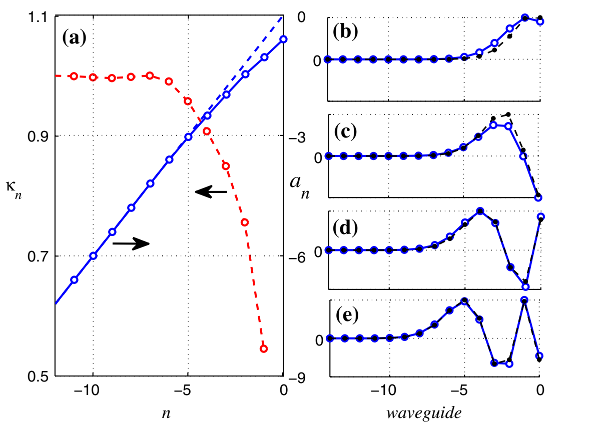

Fig. 2. (Color online) (a) Couplings (dashed line, left ordinate) and detunings (solid line, right ordinate) required for SBO in a semi-infinite WGA with $a = 0.6$, $\kappa = 1$. Dashed line is the asymptotic $a_n = 0.6n$. (b), (c), (d), (e) Mode amplitudes of SSs with eigenvalues $\lambda = 0$, $-a$, $-2a$, $-3a$, respectively (solid lines). SSs of the unperturbed array have been superposed (dashed lines).

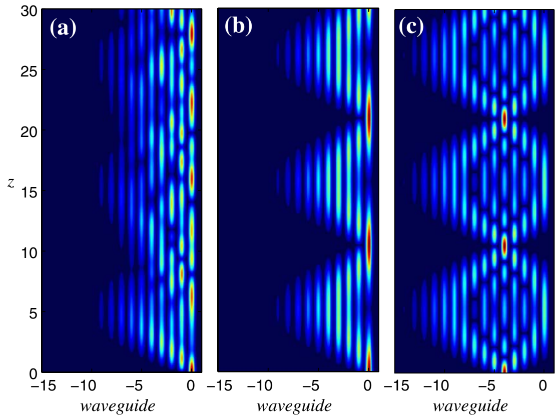

Fig. 3. (Color online) Single waveguide excitation in an (a) unperturbed and (b), (c) engineered WGA ($a = 0.6$, $\kappa = 1$).

perfect period recurrences of the excitation in contrast to the unperturbed WGA of Fig. 3(a), where revivals are impossible due to the coarse termination. As the excited waveguide moves to the left, the familiar periodic diffraction pattern of an infinite WGA [1,7] is approached [Fig. 3(c)].

As mentioned, the stronger the state localization, the less is the required extent of parameter engineering. Figure 4 shows the magnitude of the needed perturbation in detunings and couplings for various ratios a/κ. With increasing a/κ, the perturbation at a fixed site (n) decreases, and the curves drop faster with the distance from the edge. Also, as can be easily proved, in case of a negative gradient ($a < 0$), the required couplings are the same with $a > 0$, while the detunings must have the opposite deviations from the index ramp $a_n = na$. This implies that the same parameters can be used to engineer *both boundaries* of a finite WGA that is initially subject to an index ramp. Such an array supports SBO at both boundaries, in addition to standard BO in its interior, and, more generally, perfect revivals of arbitrary discrete beams. For example, in Fig. 5(a) two Gaussian beams are simultaneously launched in a relatively short WGA, whose both sides have been engineered for SBO with $a = 0.8$, while, in Fig. 5(b), three waveguides are excited with unit amplitudes in the same WGA.

We finish with some design guidelines. A practical choice is to tune the propagation constants of the

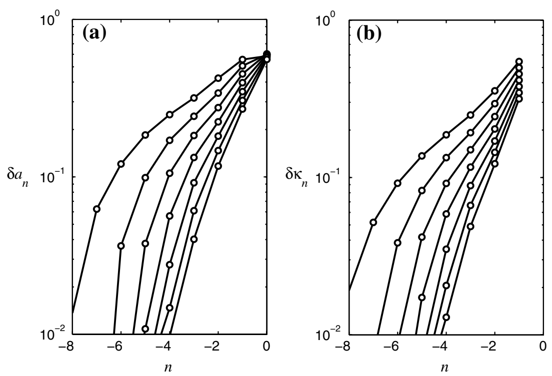

Fig. 4. Magnitude of parameter perturbations: (a) $\delta a_n = |a_n - na|$, and (b) $\delta\kappa_n = |\kappa_n - 1|$, for a increasing from 0.4 to 1.0 at 0.1 step (from left to right) and $\kappa = 1$. The abscissa is the waveguide number.

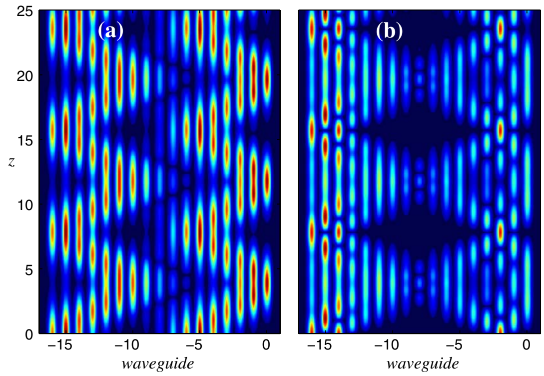

Fig. 5. (Color online) (a) SBO of two Gaussian beams in 17-element WGA that has been engineered on both sides. (b) Field evolution in the same WGA when waveguides $n = -16$, -14, -2 are excited with unit amplitudes ($a = 0.8$, $\kappa = 1$).

waveguides first, by varying their width, and subsequently, use their spacings to tune the couplings. The latter are given by the mode-overlap integral $\kappa_n = \int V_n \chi_{n+1} \chi_n^* \mathrm{d}x$, across waveguide n, where $V_n(x)$, $\chi_n(x)$ are, respectively, the potential and the power-normalized mode profile of that waveguide.

In conclusion, we have shown that a semi-infinite WGA can be engineered to support SBO. This is achieved by perturbing the involved parameters close to the edge, so that the SSs conform to the ladder of the interior WS states (or in general beat in unison with them). Such a setting is useful as a "matched" termination (or apodization) of finite WGAs with a constant index gradient to facilitate undistorted BO and perfect revivals.

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References

1. U. Peschel, T. Pertsch, and F. Lederer, Opt. Lett. **23**, 1701 (1998).
2. C. Zener, Proc. R. Soc. London Ser. A **145**, 523 (1934).
3. F. Bloch, Z. Phys. **52**, 555 (1929).
4. J. Feldmann, K. Leo, J. Shah, D. A. B. Miller, J. E. Cunningham, T. Meier, G. Von Plessen, A. Schulze, P. Thomas, and S. Schmitt-Rink, Phys. Rev. B **46**, 7252 (1992).
5. C. Waschke, H. G. Roskos, R. Schwedler, K. Leo, H. Kurz, and K. Köhler, Phys. Rev. Lett. **70**, 3319 (1993).
6. G. H. Wannier, Phys. Rev. **117**, 432 (1960).
7. T. Pertsch, P. Dannberg, W. Elflein, A. Bräuer, and F. Lederer, Phys. Rev. Lett. **83**, 4752 (1999).
8. R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. **83**, 4756 (1999).
9. G. Lenz, I. Talanina, and C. M. de Sterke, Phys. Rev. Lett. **83**, 963 (1999).
10. R. Sapienza, P. Costantino, D. Wiersma, M. Ghulinyan, C. J. Oton, and L. Pavesi, Phys. Rev. Lett. **91**, 263902 (2003).
11. C. M. de Sterke, J. N. Bright, P. A. Krug, and T. E. Hammon, Phys. Rev. E **57**, 2365 (1998).
12. H. Trompeter, W. Krolikowski, D. N. Neshev, A. S. Desyatnikov, A. A. Sukhorukov, Y. S. Kivshar, T. Pertsch, U. Peschel, and F. Lederer, Phys. Rev. Lett. **96**, 053903 (2006).
13. W. Lin, X. Zhou, G. P. Wang, and C. T. Chan, Appl. Phys. Lett. **91**, 243113 (2007).
14. N. K. Efremidis and D. N. Christodoulides, Opt. Commun. **246**, 345 (2005).