Abruptly autofocusing waves

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We introduce a new class of \((2 + 1)D\) spatial and \((3 + 1)D\) spatiotemporal waves that tend to autofocus in an abrupt fashion. While the maximum intensity of such a radial wave remains almost constant during propagation, it suddenly increases by orders of magnitude right before its focal point. These waves can be generated through the use of radially symmetric Airy waves or by appropriately superimposing Airy wave packets. Possible applications of such abruptly focusing beams are also discussed. © 2010 Optical Society of America


The focusing characteristics of optical beams have always been an issue of great practical importance [1]. In general, a wave tends to focus or defocus whenever its initial phase and/or amplitude have been suitably manipulated. In the case of a Gaussian wavefront—perhaps the most prevalent of all beams—the peak intensity follows a Lorentzian distribution around the focus or minimum waist point. Their corresponding higher-order modes (Hermite— or Laguerre—Gaussian) also behave in a similar manner [1,2]. On the other hand, the respective behavior of other families of waves can be considerably more involved, especially close to the focus.

For many applications it is crucial that a beam abruptly focuses its energy right before a target while maintaining a low intensity profile until that very moment. Ideally, this should be a linear property of the wave itself and not the outcome of any self-focusing effects [3]. In medical laser treatments this feature may be highly desirable, since the wave should only affect the intended area while leaving any preceding tissue intact [3]. In several experimental settings, such behavior can also be useful in suddenly “igniting” a particular nonlinear process, such as multi-photon absorption, stimulated Raman, and optical filaments in gases, locally only after the focus [4–7]. This will ensure that no energy is nonlinearly lost due to gradual focusing effects, for example, as is the case of Gaussian beams. To realize this fascinating prospect, it is, therefore, important to identify a new class of optical beams for which the internal energy flux tends to accumulate at the focus in an accelerated manner during propagation.

In this Letter, we introduce new families of radially symmetric (cylindrical and spherical) waves with this desired characteristic: their maximum intensity remains almost constant during propagation, while close to a particular focal point, they suddenly autofocus and, as a result, their peak intensity can increase by orders of magnitude. For a given value of the wavelength, it depends on the spatial normalization factor \(x_0\) whether paraxiality is satisfied. The propagation of an arbitrary radially symmetric initial condition \(u(r, z = 0) = u_0(r)\) can be computed in terms of the following Hankel transform pair:

\[
\frac{1}{2\pi} \int_0^\infty \kappa k \tilde{u}_0(k) J_0(kr) e^{-i\kappa z^2/2} \, dk,
\]

\[
\tilde{u}_0(k) = 2\pi \int_0^\infty \kappa r u_0(r) J_0(kr) \, dr.
\]

In the one-dimensional limit, Eq. (1) is known to support the following accelerating Airy beam [8,9]:

\[
g(x, z) = A\left(x - z^2/4 + i\alpha z\right) \exp[i(6\alpha^2z - 6i\alpha(2x - z^2) + 6ax - x^3)/12].
\]

In Eq. (4), the decay parameter \(\alpha\) ensures that the wave converges finite energy (is thus realizable) and is typically small, so that the behavior of this wave approximates in many respects [8,9] that of an ideal \((\alpha = 0)\) diffraction-free Airy wave packet [10]. Perhaps the most intriguing feature of this solution is its lateral parabolic acceleration.

Let us analyze the dynamics of radially symmetric Airy beams of the form \(u_0(r) = A\sqrt{r} e^{i(\alpha r - \alpha\theta)}\), where \(r_0\) is the initial radius of the main ring. For \(r < r_0\), the Airy beam decays exponentially, whereas the slowly decaying oscillations of the Airy tails occur outside this region. The power that the Airy beam carries is given by \(P = 2\pi \int_0^\infty |u_0(r)|^2 r \, dr = \sqrt{\pi/(2\alpha^3 r_0^{2\alpha^3/3})} (1 - 4\alpha^3)/(4\alpha)\). In the computation of the above integral, we extended the lower limit of integration from zero to minus infinity. Note that...
Fig. 1. Dynamics of radially symmetric Airy beams for \( a = 0.05, \theta_m = 10 \), and \( I_{\text{max}} = (0) = 1 \): (a) Detailed plot of the central part of the propagation dynamics. (b) Maximum intensity increases strongly with propagation distance. The wavenumber \( k = \frac{\theta_m}{r} \), which is used to define the region of autofocusing waves, is also plotted. (c) Spectral density of the initial condition for \( \theta = 0 \). The transform is a real function of \( \theta \). (d) Hankel transform of the initial condition as given by Eq. (1).
anomalous dispersion, provided that dispersion and diffraction effects are equalized. The corresponding normalized spatiotemporal equation reads

$$u_z = \left(\frac{i}{2}\right)(u_{xx} + u_{yy} + u_{tt}) = \left(\frac{i}{2}\right)(2u_r/r + u_{rr}), \tag{6}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. By applying the transformation $u(r, z) = \phi(r, z)/r$, the problem is reduced to the one-dimensional diffraction equation $i\phi_x + \phi_{rr} = 0$. Here we focus on the following exact Airy-like solution of the three-dimensional problem:

$$u(r, z) = \left[g(r_0 - r, z) - g(r_0 + r, z)\right]/r, \tag{7}$$

where $g(r, z)$ is given by Eq. (4). Note that the numerator of Eq. (7) becomes zero for $r = 0$. The first terms of the expansion of Eq. (7) close to the origin are given by

$$u(r, z) = -2g_{r_0}(r_0, z) - g_{2r_0}(r_0, z)(r^2/3), \tag{8}$$

where

$$u(r = 0, z) = -2g_{r_0}(r_0, z) = -e^{\Psi(r_0, z)}[(2\alpha + iz)\text{Ai}(\xi(r_0, z)) + 2\text{Ai}'(\xi(r_0, z))], \tag{9}$$

$$g_{2r_0}(r_0, z) = (1/2)e^{\Psi(r_0, z)}[f_1(r_0, z)\text{Ai}'(\xi(r_0, z)) + f_2(r_0, z)\text{Ai}(\xi(r_0, z))], \tag{9}$$

$$\Psi(r, z) = (6\alpha^2z - 6\alpha(2r_0 - z^2) + 6r - z^3)/12, \quad \xi(r, z) = -z^2/4 + i\alpha z + r, \quad f_1(r, z) = 2(3\alpha^2 + 4i\alpha z + r - z^2),$$

and $f_2(r, z) = 2\alpha^2 + 9i\alpha z + 6r(\alpha - z^2) + 3i\alpha - iz^3 + 2$. Equation (7) exhibits abruptly autofocusing dynamics [see Fig. 3]. However, since the energy is initially spread out on a three-dimensional ring, even higher intensity contrasts are attained; the maximum intensity that the Airy wave reaches during propagation is between 3 and 4 orders of magnitude larger than the original intensity. Other scenarios where the third dimension might be utilized include the use of two Airy pulses, the first one decelerating and the second one accelerating so that the waves simultaneously collide in space and time, or the use of appropriately chirped pulses. Such approaches are feasible even in the region of normal dispersion.

In conclusion, we have shown that families of two-dimensional and three-dimensional waves can autofocus in an abrupt fashion. We would like to point out that the general concept of autofocusing presented here is more general and, in principle, can be extended to other wave functions beyond Airy. However, the curved nondiffracting dynamics of Airy beams has several advantages, including enhanced autofocusing contrast and abruptness, especially in the case of long focal lengths. Other families of beams might also exist exhibiting abrupt autofocusing properties. In particular, we have investigated different wave configurations, the results of which will be presented elsewhere.

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References