Bloch oscillations in optical dissipative lattices

Nikolaos K. Efremidis and Demetrios N. Christodoulides

School of Optics, Center for Research and Education in Optics and Lasers, University of Central Florida, Orlando, Florida 32816

Received June 28, 2004

We show that Bloch oscillations are possible in dissipative optical waveguide lattices with a linearly varying propagation constant. These oscillations occur in spite of the fact that the Bloch wave packet experiences coupling gain and (or) loss. Experimentally, this process can be observed in different settings, such as in laser arrays and lattices of semiconductor optical amplifiers. In addition, we demonstrate that these systems can suppress instabilities arising from preferential mode noise growth.

OCIS codes: 140.3290, 130.2790.

Dissipative discrete systems such as laser arrays have been extensively studied since the late 1970s, primarily in a search to achieve high coherent powers.[1–3] In these systems, phase locking was attained by evanescent or leaky wave coupling of in-phase or out-of-phase modes. In guided configurations the refractive-index modulation between the core and the cladding regions is typically constant, and so is the local propagation coefficient across the array. Thus in the linear regime the gain preferentially amplifies a specific mode, which happens to be either the in-phase or the out-of-phase eigenvector. As a result, an input wave packet (occupying only a few waveguides) suffers from noise amplification at the low amplitude sites. Moreover, when the power is relatively low, an input beam has the tendency to spread during evolution (i.e., diffract) due to intersite coupling.

The behavior of such array systems is significantly altered when the local propagation eigenvalue linearly increases along the lattice, essentially forming a ladder. The direct analog of this setting is known to occur in solid-state physics.[4] In optics, Bloch oscillations were only recently predicted[5] and experimentally demonstrated in lossless waveguide arrays.[6,7] Naturally, one may ask whether similar processes also occur in dissipative arrays and, if this is the case, how the presence of gain and loss would affect their oscillatory behavior.

In this Letter we find that Bloch oscillations are possible in linearly ramped dissipative arrays in spite of the presence of coupling gain and loss. Physically, this type of behavior can be observed in active low-power waveguide systems, such as laser arrays or semiconductor optical amplifier lattices. In general, we find that, even though the power of the optical field tends to increase or decrease exponentially, Bloch oscillations still occur. For a specific set of parameters the total gain compensates for the loss, resulting, on average, in a constant power envelope. This is in sharp contrast with regular active arrays with zero refractive-index modulation, for which either the in-phase or the out-of-phase evanescently coupled modes (corresponding to the base and the edge of the first Brillouin zone, respectively) are preferentially amplified. In addition, we demonstrate that these systems can suppress instabilities arising from preferential mode noise growth.

To study the effect of a linear refractive-index modulation in such active arrays, the coupled mode formalism is used[8–11]:

$$i\dot{u}_n - i\epsilon u_n + \alpha(u_{n+1} + u_{n-1}) + \beta n u_n = 0\,, \qquad (1)$$

where $\dot{u}_n$ denotes a derivative with respect to time (for laser arrays) or space (for semiconductor optical amplifiers), $\alpha = \alpha_r + i\alpha_i$ is the complex coupling among successive elements of the lattice, ϵ is the linear gain or loss of the system, and β represents a linear refractive-index modulation across the lattice. We assume here that each element of the array is single mode, and thus coupling coefficient α_r among successive waveguides or cavities is positive.

Equation (1) can be derived from paraxial model $iA_z + (1/2k)\nabla^2 A + [\varepsilon_r(x, y)/2k](\omega/c)^2 A = 0$ by employing coupled-mode theory,[9] where $k = \omega(\varepsilon_r')^{1/2}/c$, $\varepsilon_r(x, y) = \Delta\varepsilon_r'(x, y) + i\varepsilon_r''(x, y)$, $(\varepsilon_r)^{1/2}$ is the refractive index of the cladding, $[\Delta\varepsilon_r'(x, y)]^{1/2}$ are variations from this value along x, and $\varepsilon_r''(x, y)$ is the gain or loss of the medium. We use a local mode expansion of the optical field $A(x, z) = \sum_n B(x - nx_0, y)u_n(z)$, where $B(x, y)$ is the mode of a single waveguide in isolation and $(nx_0, 0)$ is the center of the nth waveguide. In this case the unnormalized coupling coefficient is given by $\kappa = (1/2k)\iint B^*(x, y)\nabla^2 B(x + x_0, y)\mathrm{d}x\mathrm{d}y + (1/2k)(\omega/c)^2 \iint B^*(x, y)\varepsilon_r(x, y)B(x + x_0, y)\mathrm{d}x\mathrm{d}y$. The strengths of the real and imaginary parts of the coupling coefficient were calculated in Refs. 10 and 11. In these works it was shown that, by varying the waveguide spacing, the imaginary part of the coupling coefficient becomes both positive and negative, and its amplitude can even be greater than the real part of the coupling coefficient. We emphasize that the complex nature of the coupling constant is primarily responsible for the dissipative Bloch oscillations discussed in this Letter.

To better understand the effect of linear tilt in the refractive index, displayed in Eq. (1) by a nonzero value of β, we compare these results with the zero-tilt case. If $\beta = 0$, Eq. (1) has a complex dispersion curve that can be identified with the plane-wave solution $u_n(\theta; t) \propto \exp(ikt - i\theta n)$, where $k = k_r + ik_i$ is the complex propagation wave number and θ is the wave momentum inside the lattice. The real and imaginary parts of Eq. (1) are then found to satisfy

0146-9592/04/212485-03$15.00/0

$k_r = 2\alpha_r \cos\theta$, $g = -k_i = \epsilon - 2\alpha_i \cos\theta$, where $g(\theta)$ is the growth rate.[8] $k_r(\theta)$ represents the diffraction of the lattice. For $0 \le |\theta| < \pi/2$ the diffraction is normal, whereas for $\pi/2 < |\theta| < \pi$ the diffraction becomes anomalous. If $\theta = \pi/2$, the second-order diffraction term is zero.

On the other hand, $g(\theta)$ represents the growth of the plane-wave mode $u_n(\theta; t)$. If the gain or loss due to coupling α_i is positive, then the out-of-phase mode $[u_n(\pi; t)]$ will experience more gain (or less loss) compared with the rest of the modes and will eventually dominate. On the other hand, if $\alpha_i < 0$, the in-phase mode $[u_n(0; t)]$ takes over during evolution. For example, as we have previously shown,[8] if $\beta = 0$, the impulse response of Eq. (1) is given by $u_n(t) = BJ_n(2\alpha t)\exp(i\pi n/2)\exp(\epsilon t)$, where $J_n(x)$ (of complex argument) is a Bessel function of the first kind and of integer order n and B is the initial amplitude of u_0.

We will now consider Bloch oscillations in such dissipative lattices when β is finite. By extending the analysis of Ref. 5 in the complex domain, we find an exact solution for the impulse response of Eq. (1), i.e., $u_0 = B$ and $u_n = 0$ for nonzero n of such an infinite lattice:

$$u_n(t) = BJ_n\left[\frac{4\alpha}{\beta}\sin\left(\frac{\beta t}{2}\right)\right]\exp\left[\frac{in}{2}(\beta t + \pi)\right]\exp(\epsilon t), \tag{2}$$

where $J_n(t)$ is a Bessel function of complex argument. The argument of $J_n(t)$ oscillates in either time or space, and as a result revivals occur [apart from an amplification factor of $\exp(\epsilon t)$]. Note that gain ϵ itself leads to an overall amplification and does not participate in the process of dissipative Bloch oscillations. Apparently, the evolution of more complicated wave packets can be obtained by superimposing the response of Eq. (2). The last term in Eq. (2) indicates that, on average, the power $P = \sum_n |u_n|^2$ will increase or decrease according to $\langle P\rangle \sim \exp(2\epsilon t)$, where the average

$$\langle P\rangle = \frac{1}{T}\int_t^{t+T} P(t)\mathrm{d}t \tag{3}$$

is taken over one period of oscillations T that can be found from the first term of Eq. (2) to be

$$T = 2\pi/\beta\,. \tag{4}$$

In Fig. 1 we can see the intensity evolution arising from a single waveguide excitation, as described by Eq. (2). In particular, in the left panel of Fig. 1, $\alpha = 1 \pm i0.4$ and $\epsilon = 0$. In this case the total power is not conserved during evolution but periodically recurs every $2\pi/\beta$. The input beam initially gains power that attains a maximum at $t = T/2$. After this point, the power is reduced, and the original wave packet is restored at $t = T$. When ϵ is nonzero, the input still demonstrates similar behavior, except from an average growth in its power, as clearly demonstrated in the right panel of Fig. 1 for $\epsilon = 0.003$. Interestingly enough, the intensity profile is identical for both signs of α_i. Qualitatively, this behavior can be explained by considering the fact that the linear ramping of propagation eigenvalues causes the Bloch momentum θ to periodically scan the complex dispersion curve. As θ periodically oscillates in the Brillouin zone, the input beam experiences both normal and anomalous dispersion. In a similar way, because of the change in the value of θ, the growth rate is also periodically modulated.

Another example is depicted in Fig. 2 in which an initially in-phase Gaussian beam of the form

$$u_n = \exp\{-[(n - n_0)/A]^2\} \tag{5}$$

is launched into the lattice, where in this case $A = 2\sqrt{2}$ and $\epsilon = 0$. In the left panel, $\alpha = 1$, and thus power P remains invariant along t. In the central panel, $\alpha = 1 + i0.3$, and the $P - t$ diagram can be qualitatively explained as follows. Since the initial configuration is in-phase and $\alpha_i > 0$, the beam initially experiences losses. After some propagation length, the out-of-phase pattern grows, and the power starts to increase with t. At approximately half a period the

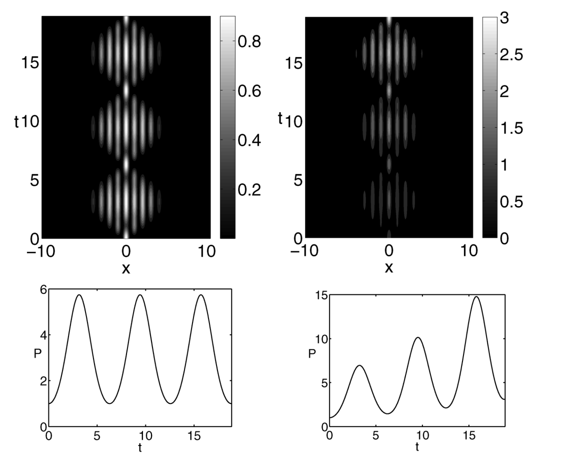

Fig. 1. Evolution arising from a single waveguide excitation when $\alpha = 1 \pm i0.4$ and $\beta = 1$. Left panel, $\epsilon = 0$; right panel, $\epsilon = 0.003$.

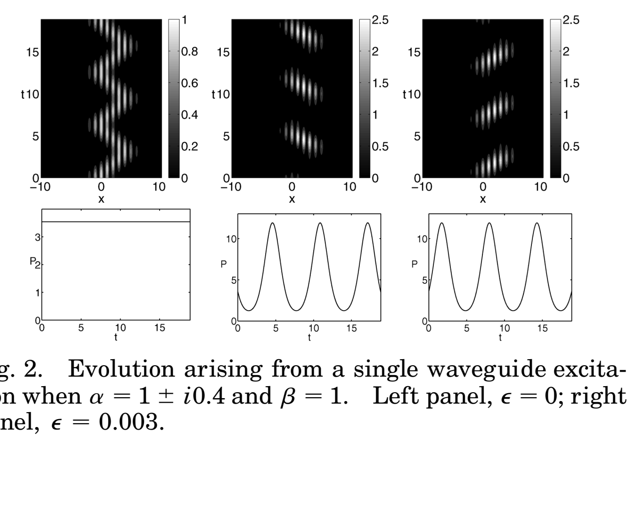

Fig. 2. Evolution arising from a single waveguide excitation when $\alpha = 1 \pm i0.4$ and $\beta = 1$. Left panel, $\epsilon = 0$; right panel, $\epsilon = 0.003$.

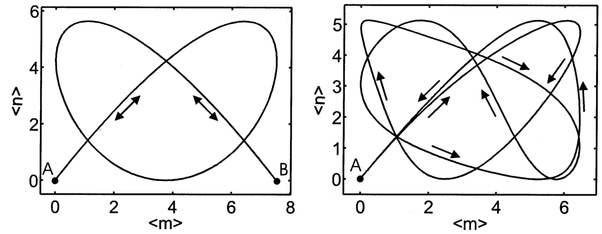

Fig. 3. Path of a beam in the $x-y$ plane when $\beta_x = 3/6$ and $\beta_y = 4/6$ and $\alpha = 1$ (left panel) and $\alpha = 1 + i0.5$ (right panel). The starting (and final) point of the oscillations is A. Note that in the conservative case the beam follows the initial path to return from B to A, whereas in the nonconservative case the beam returns to A from a different path.

phase of the pattern shifts with a tendency to become in phase, and as a result the beam experiences losses. After exactly one period, the initial wave packet is restored. The situation is reversed in the right panel in which $\alpha = 1 - i0.3$. Since $\alpha_i < 0$, the in-phase patterns have the tendency to gain power. Thus, as we can see in the $P - t$ diagram, the power initially increases with t and, eventually, after one period, the original form of the beam is restored.

The refractive-index ladder also affects the stability of the zero background. This result is of interest since it may be relevant to the study of self-confined dissipative solitons in ramped lattices. If $\beta = 0$, the only way that the zero background remains stable is if $g \leq 0$ for all values of θ, which implies that

$$\epsilon < -2|\alpha_i|. \tag{6}$$

On the other hand, if β is different from zero and due to the periodic variations of θ within the Brillouin zone, the system is stabilized when the average growth is negative,

$$g_{\text{avg}} = \int_0^{2\pi} g(\theta)\mathrm{d}\theta = 2\pi\epsilon < 0, \qquad \text{or } \epsilon < 0. \tag{7}$$

Comparison of stability conditions (6) and (7) reveals that a refractive-index ladder provides improved stability for the noise background.

Bloch oscillations are also supported in two-dimensional dissipative arrays (the conservative limit was discussed in Refs. 12 and 13). In this case the coupled-mode evolution equation is

$$i\dot{u}_{m,n} - i\epsilon u_{m,n} + \Delta_2 u_{m,n} + (\beta_x m + \beta_y n)u_{m,n} = 0,$$

where $\Delta_2 u_{m,n} = \alpha_x(u_{m+1,n} + u_{m-1,n}) + \alpha_y(u_{m,n+1} + u_{m,n-1})$. The solution of the two-dimensional model is given by a simple product of the solution given by Eq. (2), i.e.,

$$u_{m,n}(t) = BJ_m(p_x)J_n(p_y)\exp[i(mq_x + nq_y)]\exp(\epsilon t),$$

where $p_{x,y} = (4\alpha_{x,y}/\beta_{x,y})\sin(\beta_{x,y}t/2)$, $q_{x,y} = (1/2) \times (\beta_{x,y}t + \pi)$. Evidently, this system is associated with two periods, one in the x direction and one in the y direction, which are given by

$$T_x = 2\pi/\beta_x, \qquad T_y = 2\pi/\beta_y. \tag{8}$$

These oscillations are, in general, not periodic in two dimensions unless $T_x/T_y = k/l$ (rational), where k and l are nonzero relatively prime integers. Thus Bloch oscillations do not occur in two dimensions unless the ratio of the periods is a rational number. In the latter case the oscillations are periodic, and the trajectory becomes Lissajous-like with period $T = lT_x = kT_y$. Note that Bloch oscillations are not possible if either β_x or β_y is zero, since the beam will diffract along the x or y direction, respectively. To monitor the motion of the oscillating beam, we introduce the following quantity (center of mass) $\langle \mathbf{R} \rangle = \langle m \rangle \hat{x} + \langle n \rangle \hat{y}$ as

$$\langle \mathbf{R} \rangle = \sum_{m,n} (m\hat{x} + n\hat{y})\, |u_{m,n}|^2 \Big/ \sum_{m,n} |u_{m,n}|^2, \tag{9}$$

which shows the average position vector in the waveguide lattice. In Fig. 3 we see the path of the beam centroid during evolution in the case of $\beta_x = 2$ and $\beta_y = 3$.

References

1. D. Botez, in *Diode Laser Arrays*, D. Botez and D. R. Scifres, eds. (Cambridge U. Press, Cambridge, England, 1994).
2. K. Otsuka, Electron. Lett. **19,** 723 (1983).
3. J. K. Butler, D. E. Ackley, and D. Botez, Appl. Phys. Lett. **44,** 293 (1984).
4. M. Clück, A. R. Kolovsky, and H. J. Korsch, Phys. Rep. **366,** 103 (2002).
5. U. Peschel, T. Pertsch, and F. Lederer, Opt. Lett. **23,** 1701 (1998).
6. T. Pertsch, P. Dannberg, W. Elflein, A. Bruer, and F. Lederer, Phys. Rev. Lett. **83,** 4752 (1999).
7. R. Morandotti, U. Peschel, J. S. Aitchison, H. S. Eisenberg, and Y. Silberberg, Phys. Rev. Lett. **83,** 4756 (1999).
8. N. K. Efremidis and D. N. Christodoulides, Phys. Rev. E **67,** 026606 (2002).
9. H. Kogelnik, in *Guided-Wave Optoelectronics*, T. Tamir, ed. (Springer-Verlag, Berlin, 1988).
10. G. A. Wilson, R. K. DeFreez, and H. G. Winful, IEEE J. Quantum Electron. **27,** 1696 (1991).
11. H. G. Winful and R. K. Defreez, in *Diode Laser Arrays*, D. Botez and D. R. Scifres, eds. (Cambridge U. Press, Cambridge, England, 1994), pp. 226–254.
12. T. Pertsch, U. Peschel, and F. Lederer, in *Quantum Electronics and Laser Science Conference* (Optical Society of America, Washington, D.C., 1999), paper QThG8.
13. D. H. Dunlap and V. M. Kenkre, Phys. Rev. B **34,** 3625 (1986).