Spatial photonics in nonlinear waveguide arrays

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Abstract: The recent proposal of optical induction for producing nonlinear photonic lattices has revolutionized the study of nonlinear waves in waveguide arrays. In particular, it enabled the first observation of (2+1) dimensional lattice solitons, which were the first 2D solitons observed in any nonlinear periodic system in nature. Since then, progress has been rapid, with many fundamental discoveries made within the past two years. Here, we review our theoretical and experimental contributions to this effort.

OCIS codes: (230.7330) Waveguides; (130.4826) Nonlinear optics, transverse effect in; (190.5530) Pulse propagation and solitons; (999.9999) Photonic lattices

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Waveguide arrays are one- or two-dimensional (1D or 2D) photonic crystal structures. As such, they support wave dynamics equivalent to the transport dynamics of electrons in semiconductors [1]. Bragg reflections and interference effects dominate, especially for signals propagating across the array, and even linear dynamics in lattices can be fundamentally different than in homogeneous media. In the case of waveguide arrays, the diffraction relation (k_z vs. k_x, k_y) plays the role of the dispersion relation (ω vs. k) in the temporal domain. As with any periodic wave system, the linear modes in a waveguide array are extended Bloch modes, with a transmission spectrum consisting of allowed bands separated by forbidden gaps. That is, for all the spatial modes, there is a range of propagation constants which are not allowed. For a localized wavepacket, which consists of an ensemble of such modes, the band geometry determines the group dynamics. For example, a grating structure can modify the spreading of a narrow beam in much the same sense as it can affect the dispersive behavior of a temporal pulse. When such a photonic lattice is also nonlinear, propagation dynamics can be further modified. The prime example is the Kerr effect, in which the refractive index is changed as a function of the light intensity and directly depends on the spatial distribution of the light. In the case of a narrow beam, the light modifies the refractive index locally, thereby inducing a defect in the periodic structure of the lattice. Such a defect naturally has localized modes (rather than the extended Bloch modes of the ideal lattice), whose propagation constant now lies off the linear transmission band, i.e. in a gap. When these nonlinear modes induce the defect and populate it self-consistently, the wavepacket becomes self-localized and its diffractive broadening is eliminated. If this spatial profile remains constant and stable during propagation, then the beam is considered to be a "discrete soliton" or a "lattice soliton" [2]. Apart from supporting lattice solitons, nonlinearity can also couple different Bloch modes in a lattice, giving rise to such fundamental phenomena as modulation instability and spontaneous pattern formation known from homogeneous media. However, in lattices, nonlinear mode coupling can occur not only within a band but also between different bands. Novel waveguiding opportunities arise from these effects, and the potential of spatial band engineering is only starting to be explored.

The purpose of this article is to give a flavor for this potential by briefly reviewing our recent theoretical and experimental progress on spatial photonics in nonlinear waveguide arrays. Theoretically, this work will discuss the optical method of inducing waveguide arrays, the basic requirements for the observation of lattice solitons, the design of novel photonic heterostructures, and the exploitation of the different regions of band curvature for multiband vector and random-phase lattice solitons. On the experimental front, we have used the optical induction technique to provide the first demonstrations of 2D lattice solitons, spatial gap solitons in both 1D and 2D, vortex-ring lattice solitons, random-phase lattice solitons, and grating-mediated waveguiding.

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The paper is organized as follows: in Section 2, the basic physical intuition for wave propagation in waveguide arrays is given. Section 3 will cover different experimental methods of creating nonlinear waveguide arrays, focusing on the optical induction technique. Section 4 will highlight recent experimental results obtained with this method. Both novel waveguiding in the linear regime and lattice soliton formation in the nonlinear regime will be discussed. In section 5, we will briefly discuss recent theoretical ideas on multicomponent lattice solitons, focusing on multiband vector and random-phase lattice solitons. Conclusions will be given in Section 6.

2. Propagation in waveguide arrays

There are two complementary approaches to the study of waveguide arrays: an emphasis on individual waveguides that are coupled together, and an emphasis on periodic arrays whose sites have the properties of waveguides. Historically, the former approach has been predominant, with much early work focusing on discretized models with only nearest-neighbor coupling [2,3] (akin to the tight-binding approximation in solid-state physics [4]). Within this approximation the tunneling dynamics are known to lead to "discrete diffraction" [3,5], whereas in the nonlinear domain discrete solitons [2], are possible (for an overview see Ref. 6). More recently, array waveguides were also analyzed using Floquet-Bloch modes, as band theory provides a more formal description not only of the on-site modes (derived from guidance within each channel) but also of the "radiation" modes that propagate between sites. From this viewpoint, linear optical modes are described using Floquet-Bloch theory [7], with the corresponding dispersion/diffraction relation $k_z=k_z(k_x, k_y)$ characterized by bands of allowed propagation constants separated by forbidden gaps. Wave localization is accomplished through defect states: either imposed by construction, as in solid- and hollow-core photonic crystal fibers [8], or through the self-focusing effect of nonlinearity [2].

2.1 Linear propagation

A typical band structure (in **k**-space) for a linear one-dimensional waveguide array is shown in Fig. 1(a). Note that we have taken advantage of the 2π periodicity of the system and used the standard practice of representing dynamics only within the reduced (first) Brillouin zone $k_x \in (-\pi/D, \pi/D]$, where D is the lattice spacing. Each mode of the system is an extended Bloch wave with its own propagation constant (eigenvalue) and direction (normal gradient to the transmission band). During linear propagation, each mode evolves independently of the others, acquiring its own individual phase as it propagates. Any wave, or wavepacket, exciting the array is decomposed in these Bloch modes; since the modes remain the same but acquire different relative phases during propagation, the waveform may have a significantly different profile as it exits the array.

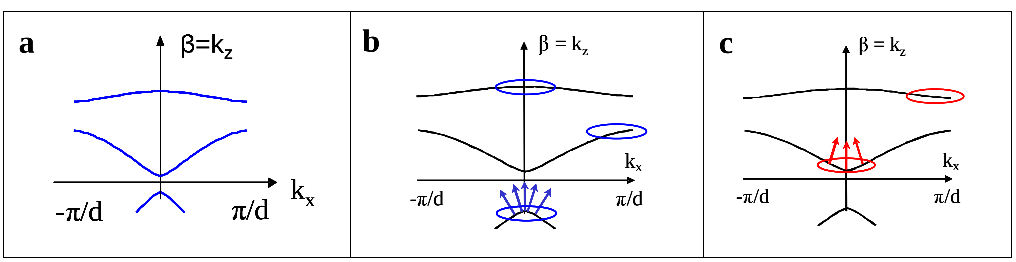

Fig. 1. Linear band structure and diffraction properties of a 1D waveguide array. (a) Transmission spectrum consisting of bands of allowed propagation constants separated by forbidden gaps. (b) Modes in convex regions experience normal diffraction. (c) Modes in concave regions experience anomalous diffraction.

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As a concrete example, consider the diffraction of a localized beam. Heuristically, the second-derivative of the diffraction relation gives the relative spread or convergence of adjacent rays, so that band curvature determines the diffractive properties of a wavepacket. More accurately, the propagation constants of the Bloch modes are governed by the transmission spectrum, so that linear dispersion is determined by the amount of relative phase acquired by the modal constituents of the beam during propagation. In regions of convex band curvature, the central mode propagates faster than its neighbors, and the beam acquires a convex wavefront during propagation (Fig. 1(b)). These are normal regions of diffraction, with wave behavior analogous to that in homogeneous media. By contrast, a group of modes in concave regions of band curvature will evolve anomalously, acquiring a concave wavefront during propagation (Fig. 1(c)). Note that there is an inflection point in each band, meaning that wavepackets propagating in this direction experience no (lowest-order) diffraction.

From an experimental perspective, simply changing the excitation angle (transverse momentum k_x relative to the array) of a beam controls its diffraction properties. This diffraction management [9] is analogous to the dispersion management of optical pulses routinely done with gratings in fibers.

Array effects on wave propagation depend on the overall size of the input beam relative to the waveguide spacing and on its internal structure. For example, a broad Gaussian beam launched on-axis into the array excites modes from different bands and stays mostly Gaussian as it propagates, as in the homogeneous case, with channels simply modulating the wave profile. By contrast, coupling a narrow beam into the fundamental guided mode of a single waveguide excites Bloch modes primarily from the first band. In this case, beam spreading is characterized by intense side lobes with little or no light in the central, starting waveguide. Known as discrete diffraction [3,5], this pattern results from coupling between channels and multiple interference effects.

2.2 Nonlinear propagation

Nonlinear effects can considerably alter propagation in a waveguide array. Bloch modes of the underlying linear lattice can undergo modulational instability, while focusing effects can counteract the diffractive tendencies of a narrow beam [2,10]. The influence of the nonlinearity, however, depends on where one is on the diffraction curve. In regions of normal diffraction, a focusing nonlinearity can compensate the convex curvature of the wavefront [2, 10], while regions of anomalous diffraction require a defocusing nonlinearity [11-14]. In turn, nonlinear dynamics can be manipulated by engineering the band structure, in much the same way as the management of linear diffraction [15].

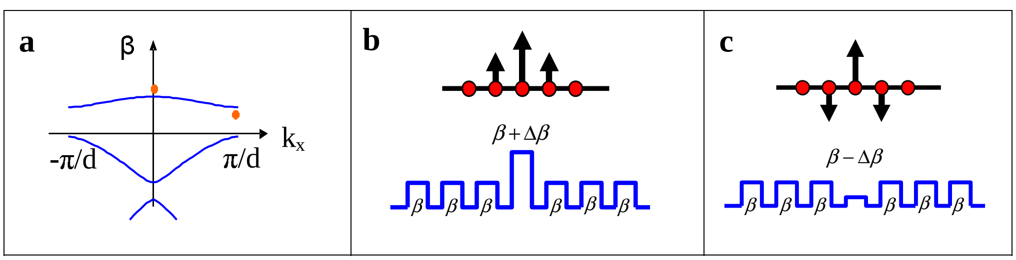

Fig. 2. 1D discrete or lattice solitons from the first band (a) 1D transmission spectrum showing the nonlinear propagation constants for the fundamental LS (base of the BZ zone) and spatial gap soliton (edge of BZ zone). (b) The fundamental LS has an in-phase structure that requires a self-focusing nonlinearity (positive defect). (c) The first-band gap soliton has a staggered phase structure that requires a defocusing nonlinearity (negative defect).

A soliton, by definition, is a nonlinear structure that maintains its spatial profile as it propagates; i.e., it is associated with a single propagation constant, as the localized

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wavepacket propagates as a single entity. In lattices, this can happen only if the propagation constant of the beam deviates from the linear bands of the transmission spectrum. That is, the propagation constant of a soliton is in a gap, and as such it represents a localized state (rather than the extended Bloch modes). For the fundamental on-axis discrete or lattice soliton [2]), the propagation constant lies in the semi-infinite gap above the first band (Fig. 2). On the other hand, the lattice soliton at the edge of the Brillouin zone [11], often called a "spatial gap soliton" [12], is formally equivalent to the spatio-temporal gap soliton observed in fibers [16-20]. Its propagation constant lies between the first and second bands, while the Bragg condition $k_x = \pi/D$ gives this soliton a staggered phase profile (Fig. 2(c)). It has no equivalent in homogeneous media. For modes originating from the first band, the curvature dictates that a defocusing nonlinearity is necessary for the formation of bright gap solitons, and the propagation constant goes down into the gap with increasing nonlinearity. For modes originating from the second band at $k_x = \pi/D$ (having a different mode structure), the propagation constant increases into the gap. As in solid-state physics, the deeper into the gap, the more localized the mode [4]. Dynamics in waveguide arrays can be richer, though, because nonlinear effects can couple defect states between bands [21] (or, equivalently, between gaps [22]).

The situation becomes more complicated with increasing dimension. Surprisingly, the requirements for higher-dimensional optical lattice solitons have only been given recently [23]. For example, band gaps always exist in one-dimensional arrays but are not guaranteed for higher-dimensional lattices [23]. A higher dimensionality also allows for points of inflection in the band curvature, for which the diffraction properties are anisotropic (even if the underlying lattice is "uniformly" periodic), and for indirect gaps, in which the minimum of one band (upper branch) occurs at a different **k**-vector than the maximum of the adjacent band (lower branch). These linear properties are illustrated in Fig. 3 for a square array. In the case of nonlinear lattices, there is a power threshold for the formation of higher-dimensional lattice solitons, a condition which does not exist for a 1D array [23].

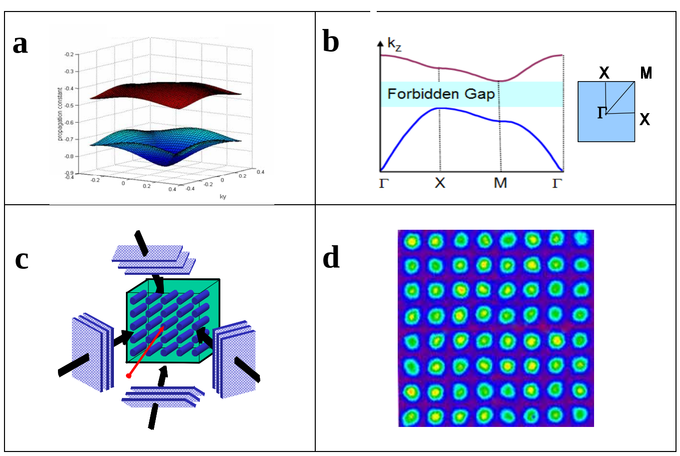

Fig. 3. Band structure and optical induction of a square waveguide array. (a,b) Band structure of a 2D square lattice, with high-symmetry points labeled in (b). (c) Interference of four plane waves to optically induce a 2D square array of 2D waveguides. (d) Photograph of a typical output face, showing a lattice with an 11μm period.

3. Methods of creating nonlinear waveguide arrays

There are currently two approaches to the experimental study of nonlinear waveguide arrays: 1D arrays of ridge waveguides etched in fixed materials, such as semiconductors [24], and the

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induction of lattices in compliant media, such as optical induction in photorefractive media [25,14,26] and periodic voltage biasing in liquid crystals [27]. Etched arrays are more stable to environmental conditions but are limited to one transverse dimension, have fixed geometries, and require pulsed lasers for nonlinear response (which often introduces spatio-temporal effects). Induced arrays, on the other hand, are sensitive to real-time conditions but do not need manufacturing to change geometries and use low-power beams that provide purely spatial dynamics. Like the ridge waveguides, voltage biasing creates a geometry that cannot be layered or extended to 2+1 dimensions. In contrast, optical induction can produce nonlinear photonic lattices in two dimensions [26] (see Fig. 3). The extra dimensions allow fundamentally new features, such as a wealth of geometries and waves with angular momentum, that are simply not available in 1D arrays. Moreover, the photorefractive materials used for the induction technique support both focusing and defocusing nonlinearities, which have allowed the observation of a broader range of lattice solitons (even in 1D).

Other methods to create two-dimensional arrays are also being developed. These include arrays of fiber bundles [28], optical fibers with multiple cores [29], and optically-written arrays in silica [30]. This latter technique has provided highly uniform linear arrays and holds much potential for the observation of nonlinear effects as well.

The optical induction technique provides a versatile approach in creating nonlinear lattices in 2D that can support lattice solitons. In this method, a periodic pattern is impressed upon a photosensitive material. Originally suggested in [25], the waveguides induced in this method can be nonlinear if the underlying medium has a nonlinear response. Quickly thereafter, basic demonstrations of this method were shown in one [14] and two [26] dimensions. These original experiments were holographic in nature, using interfering plane waves to create a periodic modulation throughout the volume of a photorefractive crystal. The basic setup is shown in Fig. 3(c). These experiments took advantage of the electro-optic anisotropy of a uniaxial Strontium Barium Niobate (SBN) crystal: the array beams were polarized in the ordinary direction, while the probe was polarized extraordinarily along the crystalline c-axis. By applying a voltage bias across this axis, the probe would feel a nonlinearity (photorefractive screening nonlinearity [31,32]), while the lattice beams would remain essentially linear during propagation. A further benefit in this system is that the sign of the nonlinearity, i.e., focusing or defocusing, simply depends on the polarity of the applied voltage.

A related induction technique is the imaging of arrays of tightly focused beams onto a photosensitive material (e.g., a photorefractive SBN crystal), but this time employing the self-focusing material nonlinearity to form an array of solitons. The array of solitons induces an array of waveguides, which serves as a nonlinear photonic lattice for other beams. This idea was initially suggested in the first experimental demonstration of permanently “fixing” (impressing) the index profile supporting a photorefractive soliton into the crystalline structure of a photorefractive crystal [33]. In that paper, it was stated explicitly that “applications include discrete solitons in 2D arrays of fixed 2D waveguides and gap solitons in periodic 2D waveguides”. However, launching arrays of solitons turned out to be very difficult because adjacent solitons interact with one another [34], thereby altering their trajectories. Consequently, parallel launched solitons do not stay parallel throughout propagation. This feature was known very early on, from the initial experiments with two solitons acting as a real-time directional coupler for a separate probe beam [35]. A partial solution to this problem was found by making the solitons incoherent with one another and launching them further apart, thereby reducing their (attractive) interaction force, and then using a probe beam of a considerably longer wavelength (to increase the coupling between the induced waveguides) [35]. However, this solution was only partially successful, and in fact the problem persisted also with arrays of permanently-fixed waveguides [36,37]. The situation has improved considerably by moving to 2D arrays of solitons, as demonstrated with

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quadratic solitons [38] and with photorefractive solitons [39], but the large separation restriction still remained: in all cases, the distance between solitons had to be larger than 30μm [38,39], to keep soliton interactions from destroying the (periodic and stationary) structure of the induced waveguide array. Naturally, such large separation implies very little coupling between sites and would mean coupling lengths in excess of 1cm, rendering experiments with photonic lattices almost meaningless because the maximum lengths of photorefractive single crystals is ~ 2cm. Some further improvement to this problem was demonstrated by Chen's group at SFSU [40], who used spatially-incoherent light passed through a mask to image their initial lattice. Using incoherent light to form the array indeed reduced the interactions among solitons. However, incoherent light diffracts faster than coherent light, and hence a higher nonlinearity is required to form the array of incoherent solitons, bringing back the problem of soliton interactions affecting the uniformity of the array. A partial way out of this problem is to sacrifice the stationary nature of the array in the propagation direction by using a smaller nonlinearity that cannot support an array of incoherent solitons [40].

Altogether, so far it is evident that the best optical induction technique to form nonlinear photonic lattices is still the one that relies on large anisotropic nonlinear response [25]. As such, this method has become the main tool for exploring solitons and nonlinear waves in two-dimensional photonic lattices. Nevertheless, the induction techniques relying on arrays of solitons, albeit being problematic for studies necessitating stationary periodic lattices, facilitate research on deformed lattices and basic science issues in elasticity, plastic motion, polaron formation [40], dynamic nonlinear lattice interactions [41-43], etc. Indeed, the ability to induce arrays, easily control the optical input, and directly image the output can give insight into a variety of other lattice problems, ranging from biology to solid-state physics to condensed matter waves in periodic traps (themselves optically-induced) [26].

From the perspective of photonic engineering, a combination of these induction techniques has recently led to the study of waveguide arrays with positive and negative defects [44], similar to solid-core and hollow-core photonic crystal fibers [8]. In these experiments, a periodic 2D array is induced using the holographic method, while a separate beam is used to overwrite or delete a set of central waveguides.

The most common material used for induction experiments is photorefractive SBN, which has a slow time response (~1s) but responds to low powers of visible light. Other photorefractive crystals, such as Indium Phosphide and Cadmium Zinc Telluride, are sensitive to IR wavelengths and hold promise to utilize the induction technique with much quicker response times (~100ns). Of course, the induction method will work for any photosensitive material of sufficient nonlinearity. For example, the technique should be possible in liquid crystal media, polymer solutions, etc. We note that the induction process has much in common with periodic traps in Bose-Einstein condensates that are effectively created using optical standing waves [45].

Most linear and self-focusing results in 1D have been obtained in fixed waveguide arrays. Linear results include the demonstrations of diffraction management [9], optical Bloch modes [46], and Bloch oscillations [47,48]. Nonlinear fixed arrays have allowed the observation of on-axis discrete solitons (i.e., at the base of the Brillouin zone) [10], interactions of discrete solitons with a single defect [49], discrete vector [50] and quadratic [51] solitons, higher-band lattice solitons [46], multiband breathers [52], and discrete modulational instability [53].

Optically-induced arrays, on the other hand, have provided experiments involving 2D lattice solitons, and all the experiments requiring a defocusing nonlinearity (e.g. bright gap solitons at the edge of the first Brillouin zone). [We note that χ^2 discrete solitons can also involve a defocusing nonlinearity]. These observations include the first demonstrations of spatial gap solitons (in both one [14] and two [26] dimensions), twisted localized lattice modes [54], fundamental [26], dipole [55], and quadrupole [55] lattice solitons in 2D, discrete vortex solitons [56,57], higher-band vortex lattice solitons [58], and random-phase solitons

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made with spatially-incoherent light [59]. It is results made possible using the optical induction technique that are highlighted here.

4. Experimental results using optically-induced photonic lattices

As mentioned above, two of the main advantages of the optical induction technique are 1) it enables the experimental study of photonic designs without the need for material fabrication and 2) it can create nonlinear lattices in 2+ dimensions, provided the underlying medium is sufficiently nonlinear. Recently, we have been exploiting these advantages in both the linear and nonlinear regimes. In either case, this involves the introduction of changes or defects into the perfect periodicity of the lattice, altering the corresponding band structure and localizing modes. In the linear case, this change is imposed during the construction of the array. In the nonlinear case, the change is a self-induced defect caused by the intensity of the propagating wave.

4.1 Grating-mediated waveguiding

As one extends the geometry of waveguide arrays into the second transverse dimension, the potential for creative lattice design greatly increases. For example, dynamically inducing positive and negative defects in a 2D lattice give wave dynamics similar to photonic crystal fibers [44], while photonic heterostructures offer opportunities for novel waveguiding. This latter case includes interfaces between different lattice types and more subtle geometries such as the quasi-2D heterostructure shown in Fig. 4. This array is periodic in one direction (**x**) but has a smoothly-varying index profile in a transverse direction (**y**) [60], and would be difficult to manufacture directly. By contrast, the optical induction technique provides a relative easy way to create and change the experimental parameters of interest, in real-time. Since this grating-mediated waveguide structure highlights the power of the induction method as well as presents a novel form of waveguiding, we briefly discuss its properties here (more details can be found in [60,61]).

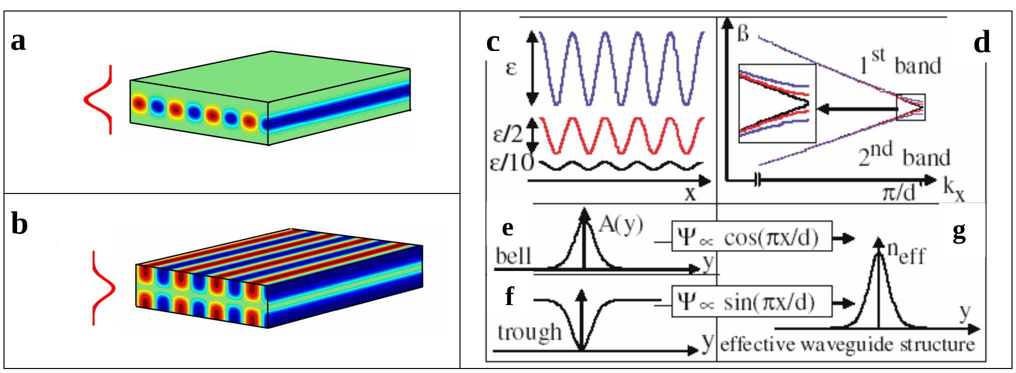

Fig. 4. Principles of grating-mediated waveguiding. (a) Type-I guide, with a bell-shaped y-profile. (b) Type-II guide, with a trough-shaped y-profile. (c) Index grating with three different amplitudes, corresponding to three different y-planes. (d) The dispersion/diffraction curves near the edge of the first Brillouin zone for gratings with index profile $n(x,y)=n_0[1+\varepsilon A(y)\cos(\pi x/D)]$. (e,f) Typical index amplitudes A(y) of (e) Type-I gratings and (f) Type-II gratings. (g) The effective waveguide structure in y: Type-I beams need a cos(πx/D) dependence and Type-II beams need a sin(πx/D) dependence to experience grating-mediated waveguiding. Taken from [60].

There are two fundamental types of grating-mediated waveguide: I) a shallow grating in **x** with a bell-shaped index profile in **y**, and II) a shallow grating in **x** with a trough-shaped profile in **y**. These index structures may be written as $n(x,y)=n_0[1+\varepsilon A(y)\cos(\pi x/D)]$, where A(y) is the amplitude profile, ε<<1, and D is the grating period. For both types of waveguide, the vertical profile is smoothly-varying on a length scale that is much larger than the period of the grating, so that the mode structure varies adiabatically in the y-direction. In this case, the band structure is almost uniform in the y-direction, with the only change occurring at the band edges; more specifically, the gaps get narrower as the waveguide depth decreases. As is shown in Figs. 4(c)-(g), this variation results in an effective waveguide index for the edge modes. The bell-shaped type I structure guides the edge mode from the first band, while the trough-shaped type II structure supports the edge mode from the second band.

Guidance in the trough-shaped structure is counter-intuitive at first, as the guiding layer is the central region with lower refractive index. To experimentally prove the principles of localization, we optically-induced the type II waveguide in a photorefractive SBN crystal (Fig. 5). Here, two plane waves interfere to form a grating in **x**, while a third narrow beam is used to overwrite the central region, creating a trough-shaped guiding layer. When the sinusoidal input mode (Fig. 5(b)) is perfectly matched in both position and phase with the grating in x, the result is mode confinement in y (Fig. 5(c)). Mismatch of either parameter, however, results in wave spreading (Figs. 5(e),(f)).

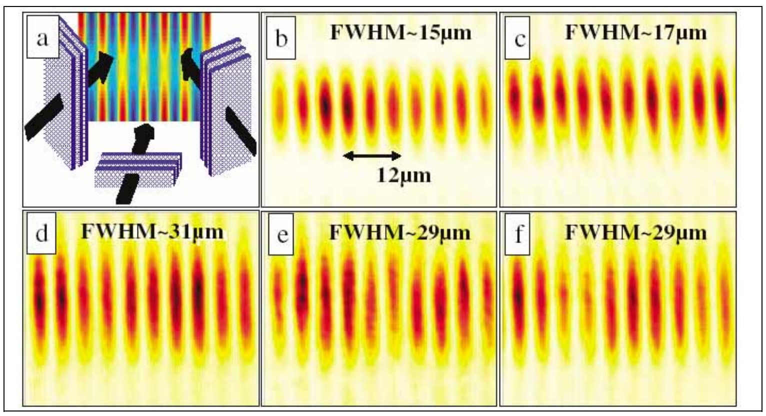

Fig. 5. Experimental scheme and resutls of a Type-II (trough-shaped) grating-mediated waveguide. (a) Schematic of optical induction of waveguide. (b) Photograph of input beam. (c) Waveguiding when position and phase of (b) match with grating structure. (d-f) Diffraction in unguided conditions: (d) Free-space diffraction of (b) in absence of waveguide. (e) Diffraction when the input is not Bragg-matched with the grating. (f) Diffraction when the input (b) is Bragg-matched but has the "wrong" phase relative to the grating. In (b-f), the intensity in each figure is normalized to its own peak intensity. Taken from [60].

A dynamical understanding of grating-mediated waveguiding can be obtained from the phase-matching condition [60]. For a structure with period D in x and amplitude variation A(y) in y, the guided mode can be approximated as $\Psi_I(x,y) = A_I(y)\cos(k_B x)$ and $\Psi_{II}(x,y) = A_{II}(y)\sin(k_B x)$, where $k_B = \pi/D$ and the subscripts indicate type I (bell-shaped) or type II (trough-shaped) waveguides. These sinusoidal waveforms can be thought of as a superposition of left- and right-going waves, each of which Bragg-reflects into the other; however, the phase change upon reflection slows the phase velocity of the coherently-added waves. This reduction is analogous to an increased effective index, and the wave is guided. Note that this mechanism is distinct from total internal reflection and confinement from traditional photonic heterostructures (in which confinement occurs in the direction of Bragg reflections, not transverse to it). That is, the grating-mediated confinement represents a fundamentally new type of waveguiding.

So far, only linear wave propagation has been studied in grating-mediated waveguides. However, their unique band structure is expected to also influence nonlinear propagation and the characteristics of solitons in these structures. Even more exciting is the self-consistent

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formation of grating-mediated structures, in which cross-phase modulation in x self-induces the transverse profile in y. This waveform, known as a holographic soliton [62], has not yet been demonstrated experimentally.

4.2 Lattice solitons

Lattice solitons result from a balance between coupling among adjacent waveguides and nonlinear focusing effects [2]. The simplest example is the case of on-axis input at the base of the Brillouin zone (Figs. 2(a),(b)), first demonstrated in fixed AlGaAs arrays [10] The observation of this soliton in optically-induced arrays [14], shown in Fig. 6, was used to demonstrate the principle and effectiveness of the induction technique. In the linear regime, a probe coupled straight into a single waveguide channel experiences discrete diffraction (Fig. 6(a)). Nonlinearly, the light intensity collapses into a lattice (discrete) soliton (Fig 6(b),(c)). For probe beams with zero transverse momentum, Bragg reflections from the array are minimized, and light in each channel is in-phase with its neighbors [2].

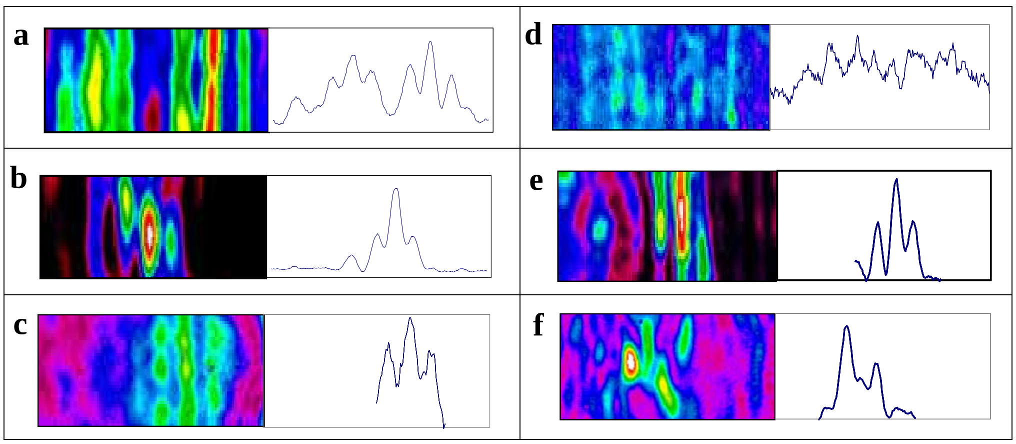

Fig. 6. Experimental observation of discrete diffraction and solitons in 1D optically-induced waveguide arrays. (a-c) Propagation for an on-axis input probe. Discrete diffraction (a) in the linear regime. Soliton intensity (b) and relative phase (c) in the nonlinear regime (+1.6kV/cm). The phase information (c) is obtained by interfering the output signal (b) with a plane wave. (d-f) Propagation at the Bragg angle, corresponding to the edge of the first Brillouin zone. Discrete diffraction (d) in the linear regime. Soliton intensity (e) and relative phase (f) in the nonlinear regime (-2.0kV/cm). Note that the central peak experiences destructive interference, while the surrounding lobes experience constructive interference. Note also the need for defocusing nonlinearity (negative voltage) to create the 1D gap soliton (e). Taken from [14].

By contrast, probe beams angled at the edge of the Brillouin zone experience the maximum Bragg effects. A combination of scattering and nonlinearity couples the underlying Bloch modes [63], and a spatial gap soliton can form under the appropriate conditions [11,12,14]. As described above, there are two types of bright discrete soliton with propagation constant in the first band gap: those arising from the first band, requiring a defocusing nonlinearity, and those arising from the second band, requiring a focusing nonlinearity. In both cases, the definition of the Bragg angle $k_x = \pi/D$ (where D is the lattice period) assures that the wavefunction in each waveguide is π out-of-phase with its neighbors [11]. The first bright spatial gap soliton to be observed was the first-band case, demonstrated by our group in optically-induced photonic lattices [14] (we note that evidence for the dark gap soliton from the first band was reported earlier in [13]). Reproduced in Fig. 6(e), the intensity profile is that of an on-axis discrete soliton while its relative phase, obtained by interfering the soliton output with a plane wave, shows a clear π phase difference between adjacent sites (Fig. 6(f)). (Hence, we originally called it a "staggered soliton" after its characteristic phase profile [14].) Shortly thereafter, spatial gap solitons were demonstrated in fixed 1D arrays with self-focusing nonlinearity [46, 64]. We note that the initial observations excited the gap solitons using a single angled beam input [14, 46], letting Bragg reflections generate the counterpropagating component, while subsequent work showed that a two-beam (sinusoidal) input, as originally suggested in [12], provided more efficient generation of the solitons [65, 66].

More complicated lattice or discrete soliton structures, with different mode profiles and internal components, have also been observed in 1D arrays. These include the dipole-mode lattice soliton [54], the vector lattice soliton [50], in which two or more components need to be present simultaneously for soliton formation (otherwise propagation is not stationary, e.g. diffracting or breathing), and the discrete quadratic soliton, consisting of a fundamental frequency and its second harmonic [51]. Interactions between solitons in 1D are also starting to be observed. In particular, the 1+1D collision between a moving "signal" beam and a stationary "blocker" has been experimentally demonstrated [67], paving the way towards routing, switching, and logic operations using discrete spatial solitons [68].

While such collisions hold more promise in 2D geometries, so far only single soliton structures have been observed in 2D photonic lattices. Made possible by the optical induction technique, the first such 2D structures to be observed were the fundamental on-axis and spatial gap solitons (shown in Fig. 7) [26]. These solitons represent the first demonstrations of their type for any physical system.

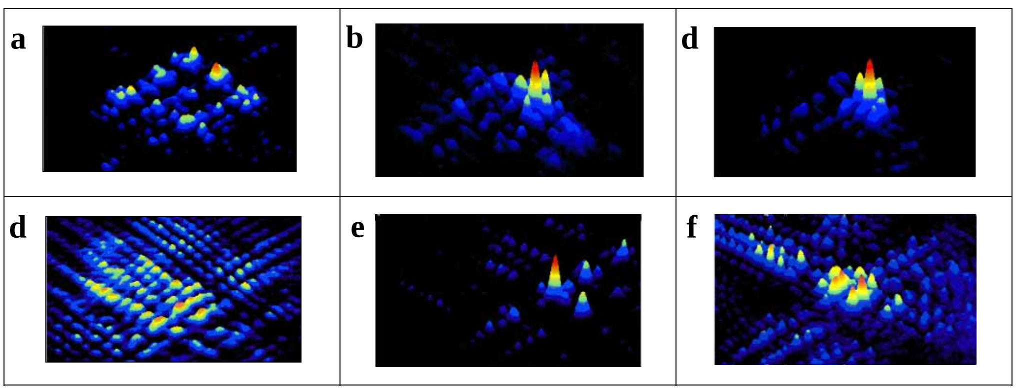

Fig. 7. Experimental observation of discrete diffraction and solitons in 2D optically-induced waveguide arrays. (a-c) Propagation for an on-axis input probe. Discrete diffraction (a) in the linear regime. Soliton intensity (b) and relative phase (c) in the nonlinear regime (+1.6kV/cm). The phase information (c) is obtained by interfering the output signal (b) with a plane wave. (d-f) Propagation at the Bragg angle, corresponding to the corner of the first Brillouin zone (M-point in Fig. 4c). Discrete diffraction (d) in the linear regime. Soliton intensity (e) and relative phase (f) in the nonlinear regime (-1.6kV/cm). Note that the central peak experiences destructive interference, while the surrounding lobes experience constructive interference. Note also the need for defocusing nonlinearity (negative voltage) to create the 2D gap soliton (e). Taken from [26].

One of the advantages of higher-dimensional lattices is that waves with angular momentum can be studied. Vortex motion in 2D periodic systems is universal yet nontrivial: the discrete set of sites breaks the rotational symmetry necessary to guarantee the conservation of angular momentum, while reflections from Bragg planes complicate the rotating wave dynamics. In practice, however, the vortex may be treated as a pair of "twisted localized modes," [69,70], and the lattice potential preserves the relative phase between sites [69, 71]. In the case of solitons, bright vortex rings are possible under self-focusing nonlinearity [69,71]. This is in distinct contrast to the homogeneous case, where bright rings

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are unstable [72] and only dark vortex solitons are possible under defocusing nonlinearity [73].

There are two types of fundamental (single-charge) vortex soliton: on-site solitons [69, 56], in which the singularity is centered on a waveguide site, and off-site solitons [71,56,57], in which the singularity is centered between sites. In contrast with the 1D analogues of the (stable) Sievers-Takeno [74] and (unstable) Page [75] modes, both of the basic discrete vortex types are stable. Examples of such vortex behavior in a lattice, taken from [56], are shown in Fig. 8**.** In the linear regime, the vortex simply diffracts across the lattice, maintaining its characteristic spiral phase structure (Figs. 8(a),(b)). When the nonlinearity is present, both on-site (Figs. 8(c),(d)) and off-site (Figs. 8(e),(f)) vortex lattice solitons can form.

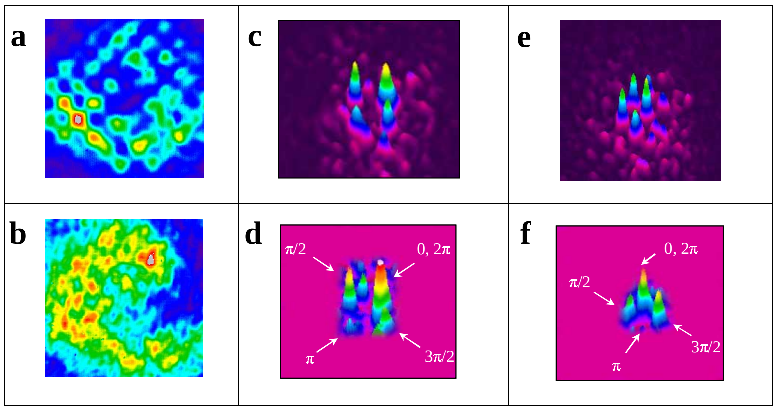

Fig. 8. Experimental observation of discrete vortex solitons in 2D optically-induced waveguide arrays. (a) Discrete diffraction and (b) spiral phase structure (formed by interfering the output (a) with a plane wave) in the linear regime. (b) Soliton intensity and (c) relative phase in the nonlinear regime (+1.6kV/cm) for a vortex with its singularity centered on-site. (d) Soliton intensity and (e) relative phase in the nonlinear regime (+1.6kV/cm) for a vortex with its singularity centered in-between sites. The vortex solitons keep their relative phase, despite the fact that the conservation of angular momentum (topological charge) is not guaranteed in a lattice. Taken from [56].

Naturally, richer vortex soliton structures are also possible in a lattice. For example, asymmetric [76] and higher-charge [69,77] vortex lattice solitons have been predicted, while higher-band vortex lattice solitons have both been predicted [78] and recently observed [58]. All of these vortex-ring solitons are generic to nonlinear lattices in two dimensions and are among the building blocks for more extended and complicated wave structures. The vortices shown in Fig. 8 are the most basic, and it is anticipated that such solitons will be observed in other systems, such as nonlinear fiber bundles, photonic crystal fibers, and Bose-Einstein condensates, in the near future.

5. Multi-component, multi-band vector, and random-phase lattice solitons

Vector solitons are multi-component nonlinear structures that require the presence of all the components to maintain their steady-state profile. They range from the simplest case of just two components, each of which would diffract without the presence of the other [79], to the more complicated nonlinear dynamics of a statistical ensemble of modes [80]. Discretized versions of the first-type were proposed in [81], who considered the joint interaction of modes from the first band. However, in lattices there is the additional element of the underlying band structure, with different regions of curvature (and hence different reactions to a given nonlinearity) and the possibility of components coming from different bands. In these cases, including the more generalized example of random-phase lattice solitons, a continuous model is needed [e.g., 25, 82]

The first experimental demonstrations of vector lattice solitons were for components contained within a single band, displaying the jointly-trapped propagation of two fundamental (on-axis) discrete solitons [50,83]. More complex structures, such as vector lattice solitons with modes arising from different bands [21] (or, equivalently, components residing in different gaps [22]), including vector modes with vorticity [78], have been studied theoretically but not yet observed (though discrete multiband breathers, exhibiting non-stationary propagation, were reported in [52]). In these cases, it is crucial that the components all come from regions with the same type of curvature. For example, waveguides with a self-focusing nonlinearity require that components arise from concave regions of the transmission spectrum. Note that *all* the components combine to induce a single defect state in which they are *jointly* trapped.

The prediction of multi-band vector solitons [21] paved the way to the discovery of random-phase (or incoherent) lattice solitons [82]. These latter solitons contain stochastically fluctuating fields, whose time-averaged envelope induces a defect state in the lattice. The underlying intuition is as follows. The incoherent beam of light is decomposed into an ensemble of modes, whose population is stochastically fluctuating. The time-averaged intensity of this random-phase ensemble is localized in space, and through the nonlinearity induces a broad defect in the lattice. In turn, this induced defect has many bound (localized) states, and the random-phase soliton forms when the time-averaged modal population is self-consistent with the defect these modes jointly induce [82]. A crucial condition for the formation of such random-phase lattice soliton is therefore that the nonlinearity responds only to the time-averaged intensity, a condition that is identical to the one enabling incoherent solitons in homogeneous media [e.g., 80]. As in the case of basic vector lattice solitons, the modal components of a random-phase lattice soliton must all come from regions of the same curvature (see Fig. 9). Consequently, the soliton power spectrum, showing the probability distribution of modes, takes on a characteristic multi-humped profile of random-phase ensembles of modes, arising from different bands but all experiencing the same type of diffraction (normal or anomalous) [82]. Note that the k-vector cutoffs are sharp in the Floquet-Bloch basis appropriate to lattices but are more rounded in the Fourier representation.

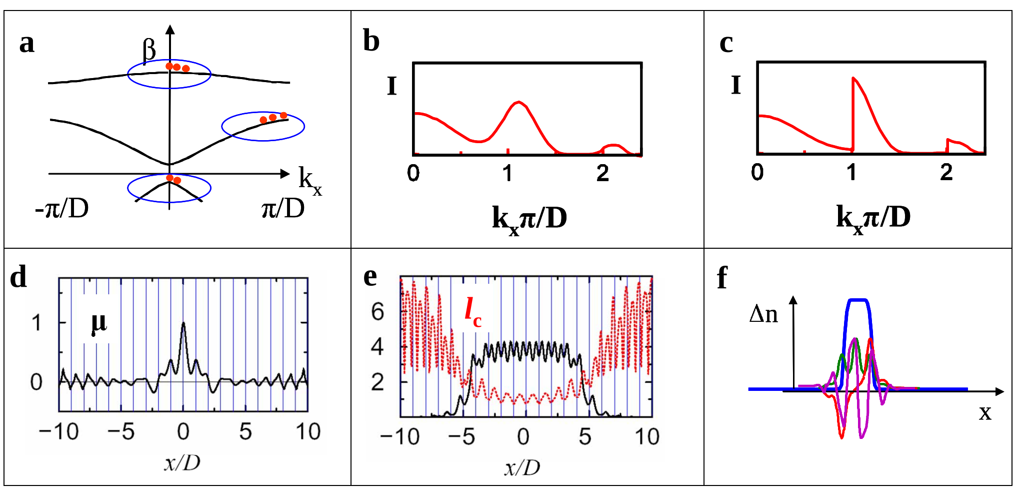

Fig. 9. Theoretical characterization of random-phase lattice soliton (RPLS). (a) Modes are taken statistically from regions of the transmission spectrum with the same band curvature. (b,c) Power spectrum in the (b) Fouier basis and (c) Floquet-Bloch basis. Note the sharp cut-offs in (c), implying that the F-B representation is more appropriate for lattice modes. (d,e) Statistical properties of RPLS. The blue lines indicate the centers of each waveguide in the 1D array. (d) Mutual coherence/correlation function centered on the central waveguide. (e) Soliton profile in black and correlation length in red. (f) Modal structure as given by waveguide theory. See text for its relation to the correlation behavior in (e). Taken from [82].

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Recently, such random-phase lattice solitons have been demonstrated using spatially-incoherent light in 2D waveguide arrays [59]. The experiments of [59] revealed that the multi-humped power spectrum occurs even for an initial distribution that is homogeneous in k-space. That is, there is a nonlinear energy exchange that transfers power into those regions of curvature that experience focusing under the given nonlinearity. In a sense, the internal dynamics are complementary to the famous Fermi-Pasta-Ulam problem of energy distribution in nonlinear lattices [84].

The spectral filtering in nonlinear lattices has interesting effects on the statistical properties of the light. In particular, despite the partial coherence of a random-phase lattice soliton, all its properties (e.g., its intensity structure, statistical properties, and power spectrum) must conform to the lattice periodicity. As a specific example, consider the mutual correlation function $\mu(x,x') = B(x,x')/\sqrt{I(x)I(x')}$, where $B(x,x') = \sum_n c_n \psi_n(x)\psi_n(x')$ is a measure of the light statistics, and the intensity I(x) = B(x,x). $\mu(x,x')$ is shown in Fig. 9(d), and the related correlation length $l_c(x) = \int dx' |\mu(x,x')|^2$ is shown in Fig. 9(e). In these figures, the blue lines indicate the centers of the waveguides. Within the soliton profile (Fig. 9(e)), one finds that μ maintains its profile as one moves left and right across the array; that is, μ(x,x') ≈ μ(x+D,x'+D). Similarly, the spatially-dependent correlation length l_c(x) has a periodic behavior within the soliton. This periodicity is reminiscent of Bloch's theorem for functions in a linear lattice, but the nonlinearity present here indicates that more subtle effects are at play.

The correlation length l_c(x) has an interesting profile with respect to the soliton (Fig. 9(e)). Within the soliton, l_c is relatively low, with minima at the waveguide centers and maxima between sites. Outside the soliton, the correlation increases and takes on a more complex structure. Both of these features can be explained using the intuition described above and shown in Fig. 9(f). In the vicinity of the induced defect, many defect modes (localized states) coexist, with randomly-varying phase, thereby reducing the correlation. Away from the induced defect, on the other hand, these modes decay at different rates: the higher the mode, the larger its propagation constant (Fig. 9(a)), and the slower it decays away from the induced defect. Far away from the defect, only a few modes survive (the highest modes contributing to the soliton; i.e., the modes from the highest band). Since a mode is always correlated with itself, l_c(x) goes up with increasing distance from the soliton.

To conclude this section, the recent discovery of random-phase lattice solitons [82,59] is perhaps one of the most fundamental discoveries made with nonlinear periodic structures. The recent observation of [59] constitutes the first observation of a random-phase soliton in any nonlinear periodic system in nature and has implications far beyond optics – basically to any nonlinear periodic system in which stochastic waves can propagate, suggesting random-phase solitons in quasi-thermal matter waves [85], plasmas, sound, etc. For applications, this observation paves the way for the k-space spectroscopy of Brillouin zones [44] and for the study of white light propagation in photonic lattices.

6. Conclusions

This article has presented a brief review of spatial photonics in nonlinear waveguide arrays, focusing on new band-theoretical results and on recent 2+1D observations made possible by the optical induction of photonic lattices. The emphasis has been on beam dynamics, interpreting the competition between waveguide coupling and nonlinear localization in terms of the band/gap transmission features familiar from the theory of periodic systems. In the linear case, defect states are imposed by construction, such as the band engineering that led to grating-mediated waveguiding [60]. In the nonlinear case, localized modes and lattice solitons act as self-induced defect states, with propagation constants lying in the gaps between (linear) bands. Depending on the type of local diffraction, discrete soliton formation may require focusing or defocusing nonlinearity. Vector lattice solitons are also possible, with the condition that their constituent components must all come from regions of the same band curvature [21]. Stochastic lattice solitons, in which the phase front is randomly-varying, have been predicted [82] and observed [59]. In addition to providing the diagnostic technique of Brillouin zone spectroscopy [44], these studies give insight into a wide range of partially-correlated dynamics in nonlinear periodic systems (e.g., partially-condensed matter waves [85]).

The optical induction of waveguide arrays has opened the door to the *experimental* study of nonlinear lattice dynamics in two transverse dimensions, and it has allowed the first observations of many nonlinear lattice structures that were hidden in other fields (e.g., 2D lattice solitons [26], discrete vortex solitons [56,57], and random-phase lattice solitons [59]). As the technique is essentially holographic, it also provides the exciting opportunity to design and test waveguide geometries that are difficult to manufacture directly. These include arrays with defects [44], interfaces between lattices, and novel photonic heterostructures (e.g., [60]). One dimensional arrays in 2D environments would allow for multidimensional waveguide coupling and junctions [86], leading the way towards discrete soliton routing and network applications [68]. As with photonic crystal fibers, the geometrical freedom that two-dimensional arrays can provide will undoubtedly lead to more new and exciting results.

Indeed, the theme of this work has been the ability of waveguide arrays to act as spatial photonic structures. These lattice geometries provide new environments for traditional nonlinear optics as well as opportunities for fundamentally new types of behavior. In the case of nonlinear waveguide arrays, basic structures such as lattice solitons have been observed, but the field is still in its infancy. While the foundation has been laid, the potential of spatial photonic band engineering is only starting to be explored.

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