

## Homework #4

**Due date:** December 19, 2006

### Notes:

1. Please write your name on the homework you are going to hand in.
2. Homeworks are to be solved and written individually. Any form of copying or plagiarism is prohibited.
3. This homework is to be handed in the latest by the beginning of the class on December 19th, that is by 15:15. Late homework will not be accepted.
4. In case you have any questions send email to the class mailing list:  
[em201-list@tem.uoc.gr](mailto:em201-list@tem.uoc.gr)

**Problem 1 [30 points]** Determine the discrete arithmetic function that corresponds to each one of the following generating functions:

(a) [10 points]

$$A(z) = \frac{1 + z^2}{4 - 4z - z^2}$$

(b) [10 points]

$$A(z) = \frac{1}{1 - z^3}$$

(c) [10 points]

$$A(z) = \frac{1}{(1 - z)(1 - z^2)(1 - z^3)}$$

**Problem 2 [10 points]** Let

$$a_n = \sum_{i=0}^n i^2.$$

(a) [5 points] Show that  $a_n$  is  $O(n^3)$ .

(b) [5 points] Show that  $a_n$  is  $n^3/3 + O(n^2)$ .

**Problem 3 [20 points]** Solve the following recurrence relation

$$a_n - 5a_{n-1} = 3^n + 4 \cdot 5^n, \quad n \geq 1,$$

given that  $a_0 = 2$ .

**Problem 4 [20 points]** Solve the following recurrence relation

$$a_n - 28a_{n-1} + 187a_{n-2} = f(n), \quad n \geq 2,$$

where

$$f(n) = \begin{cases} 11, & n = 11 \\ 17, & n = 17 \\ 0, & \text{otherwise} \end{cases},$$

and given that  $a_0 = 11, a_1 = 17$ .

**Problem 5 [20 points]** Solve the following recurrence relation

$$3a_n^2 - a_{n-1}a_{n-2} = 0, \quad n \geq 2,$$

given that  $a_0 = 9, a_1 = 81$ .

**Total points: 100**