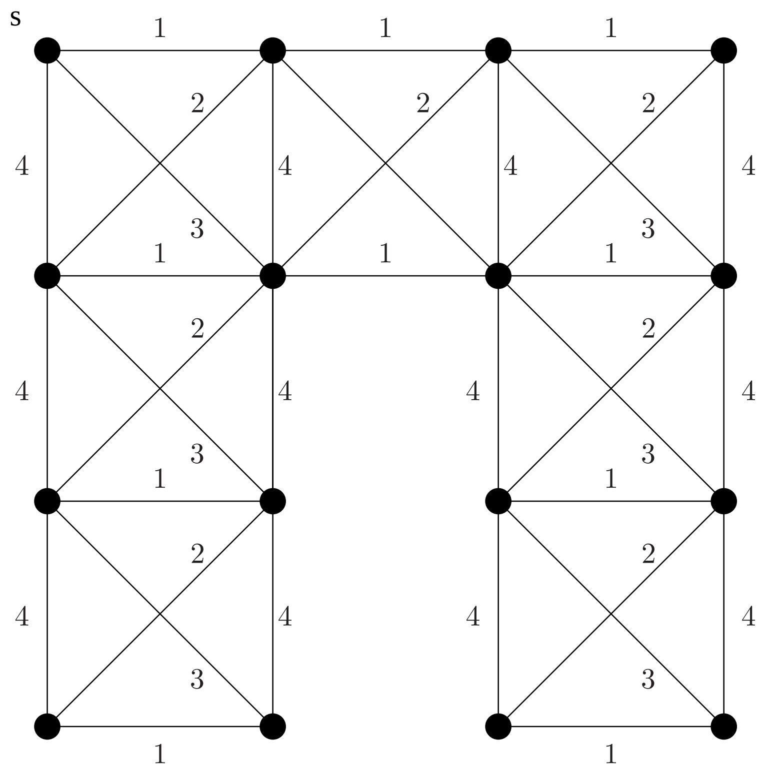
Homework #3

Due date: December 5, 2006

Notes:

1. Please write your name on the homework you are going to hand in.
2. Homeworks are to be solved and written individually. Any form of copying or plagiarism is prohibited.
3. This homework is to be handed in the latest by the beginning of the class on December 5th, that is by 15:15. Late homework will not be accepted.
4. In case you have any questions send email to the class mailing list: `em201-list@tem.uoc.gr`

Problem 1 [20 points] Compute the shortest paths from vertex s to every other vertex of the following undirected weighted graph. Which are the lengths of the shortest paths you found?



Problem 2 [20 points] Let $G = (V, E)$ be an undirected graph, with $|V| \geq 2$. Consider the graph $G' = (V', E')$, constructed by adding a vertex u to G and connecting u to every vertex in G. In other words, $V' = V \cup \{u\}$ and $E' = E \cup \{(u, v) \mid v \in V\}$. Answer the following questions and justify your answers:

(a) [10 points] If G has an Euler path, does G' also have an Euler path?

(b) [10 points] If G has an Euler cycle, does G' also have an Euler cycle?

Problem 3 [20 points] An ordered n-tuple $(d_1, d_2, \ldots, d_n)$ of non-negative integers is called *graphable*, if there exists a linear undirected graph without loops, with n vertices, the degrees of which are $d_1, d_2, \ldots, d_n$.

(a) [5 points] Show that $(4, 3, 2, 2, 1)$ is graphable.

(b) [5 points] Show that $(3, 3, 3, 1)$ is not graphable.

(c) [10 points] Without loss of generality, assume that $d_1 \geq d_2 \geq \ldots d_n$. Show that the n-tuple $(d_1, d_2, \ldots, d_n)$ is graphable if and only if the $(n-1)$-tuple $(d_2 - 1, d_3 - 1, \ldots, d_{d_1} - 1, d_{d_1+1} - 1, d_{d_1+2}, \ldots, d_n)$ is graphable.

Problem 4 [20 points] A linear planar graph without loops is called a *triangulation*, if every face, except the infinite one, has exactly three edges, and if it is not possible to add an edge between two vertices of the graph, while remaining planar. Consider a triangulation T with $n \geq 5$ vertices, the infinite face of which has exactly 5 vertices. Compute the number of edges of T as a function of its number of vertices, i.e., as a function of n.

Hint: Find a relation between the number of faces of T and the number of edges of T.

Problem 5 [20 points] A graph is called *self-complementary* if it is isomorphic with its complement.

(a) [5 points] Find a self-complementary graph with four vertices.

(b) [5 points] Find a self-complementary graph with five vertices.

(c) [10 points] Show that every self-complementary graph G has either $4k$ or $4k + 1$ vertices.

Hint: First observe that the sum of the degree of the vertices of a graph is equal to two times the number of its edges (why?). Then compute the number of edges of G as a function of the number of vertices of G, using the fact that G and its complement are isomorphic graphs.

Total points: 100