Homework #1

Due date: October 31, 2006

Notes:

1. Please write your name on the homework you are going to hand in.
2. Homeworks are to be solved and written individually. Any form of copying or plagiarism is prohibited.
3. This homework is to be handed in the latest by the beginning of the class on October 31st, that is by 15:15. Late homework will not be accepted.
4. In case you have any questions send email to the class mailing list: em201-list@tem.uoc.gr

Problem 1 [20 points]

(a) [10 points] Prove that if A and B are infinite countable sets, then their Cartesian product $A \times B$ is also a countable set.

(b) [10 points] Use the above result of to show inductively that the set $\mathbb{N}^k$, $k \geq 2$, where

$$\mathbb{N}^k = \begin{cases} \mathbb{N} \times \mathbb{N}, & k = 2 \\ \mathbb{N} \times \mathbb{N}^{k-1}, & k > 2 \end{cases},$$

is a countable set.

Problem 2 [20 points] Construct the truth tables for the following statements:

(a) [5 points] $(p \to p) \vee (p \to \overline{p})$

(b) [5 points] $(p \vee \overline{q}) \to \overline{p}$

(c) [5 points] $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

(d) [5 points] $(\overline{q} \to \overline{p}) \to (p \to q)$

Problem 3 [15 points] From a set of 210 students, 90 wear a hat in class, 71 wear a scarf and 50 wear a hat and a scarf. From the 84 students that wear a sweater, 46 wear a hat, 41 wear a scarf and 32 wear a hat and a scarf. All students that wear neither a hat nor a scarf wear gloves.

(a) [5 points] How many students wear gloves?

(b) [5 points] How many students that do not wear a sweater, wear a hat but not a scarf?

(c) [5 points] How many students that do not wear a sweater, wear neither a hat nor a scarf?

Problem 4 [20 points] Define a relation R over the set of all positive odd integers such that

$$R = \{(a, b) \,|\, a - b \text{ is an odd positive integer}\}.$$

Is R reflexive? Symmetric? Antisymmetric? Transitive? Is it an equivalence relation? Is it a partial order?

Problem 5 [25 points] Let R be a relation defined over A. Let $R_1, R_2, \ldots, R_i, \ldots$ be the successive transitive extensions of R. Prove using induction that if (a, b) belongs to R_i (for some $i \geq 1$), then there exist n elements in A, $n \leq 2^i - 1$, $x_1, x_2, \ldots, x_n$, such that $(a, x_1), (x_1, x_2), \ldots, (x_{n-1}, x_n), (x_n, b)$ all belong to R.

Total points: 100