Theoretical Micromagnetics

Lecture Series

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Lecture 3a. Dynamical equation for the magnetization

The Landau-Lifshitz equation

We derive equations for the dynamics of the magnetization as Hamilton's equations obtained for the magnetic energy E,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \boldsymbol{F}, \qquad \boldsymbol{F} = -\frac{\delta E}{\delta \mathbf{M}}.$$

Remark

The dynamical part of the Landau-Lifshitz equation has the same form as the equation of motion of a magnetic moment in a field.

It conserves the length of the local magnetization vector

$$\frac{d}{dt}\left(\mathbf{M}^2\right) = -2\gamma \mathbf{M} \cdot (\mathbf{M} \times \boldsymbol{F}) = 0.$$

A conservative model

Hamilton's equations

The Landau-Lifshitz equation is Hamilton's equation where the magnetic energy is the Hamiltonian. It conserves the energy

$$\frac{dE}{dt} = \int \left(\frac{\delta E}{\delta \mathbf{M}} \cdot \dot{\mathbf{M}} \right) d^3x = 0$$

Rationalized form of effective field

Assume exchange, anisotropy and external field energy (in 1D)

$$E = \frac{A}{M_s^2} \int \partial_x \mathbf{m} \cdot \partial_x \mathbf{m}\, dx + \frac{K}{M_s^2} \int (M_s^2 - M_3^2)\, dx - \mu_0 \int \mathbf{H} \cdot \mathbf{M}\, dx.$$

We find

$$\boldsymbol{F} = -\frac{\delta E}{\delta \mathbf{M}} = \frac{2A}{M_s^2} \partial_x^2 \mathbf{M} + \frac{2K}{M_s^2} M_3 \hat{\boldsymbol{e}}_3 + \mu_0 \mathbf{H}$$

$$= \mu_0 \left(\frac{2A}{\mu_0 M_s^2} \partial_x^2 \mathbf{M} + \frac{2K}{\mu_0 M_s^2} M_3 \hat{\boldsymbol{e}}_3 + \mathbf{H} \right) = \mu_0 M_s \left(\ell_{\rm ex}^2 \partial_x^2 \mathbf{m} + k^2 m_3 \hat{\boldsymbol{e}}_3 + \mathbf{h} \right)$$

We rescale space $x \to x\, \ell_{\rm ex}$ and we have

$$\boldsymbol{F} = \mu_0 M_s \boldsymbol{f}, \qquad \boldsymbol{f} = \partial_x^2 \mathbf{m} + k^2 m_3 \hat{\boldsymbol{e}}_3 + \mathbf{h}.$$

Rationalized form of the equation

The Landau-Lifshitz equation is $\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \boldsymbol{F}$, or

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \boldsymbol{F} \Rightarrow \frac{\partial \mathbf{m}}{\partial t} = -(\gamma \mu_0 M_s) \mathbf{m} \times \boldsymbol{f}, \qquad \boldsymbol{f} = -\frac{\delta E}{\delta \mathbf{m}}.$$

Rescale time $\tau = (\gamma \mu_0 M_s) t$.

The Landau-Lifshitz equation in rationalized form

$$\frac{\partial \mathbf{m}}{\partial \tau} = -\mathbf{m} \times \boldsymbol{f}.$$

- Lengths are measured in units of $\ell_{\rm ex}$.
- Time is measured in units of $1/(\gamma \mu_0 M_s)$.

We define the energy (that gives the desired effective field)

$$E = \frac{1}{2} \int (\partial_x \mathbf{m} \cdot \partial_x \mathbf{m}) dx + \frac{k^2}{2} \int (1 - m_3)^2 dx - \int \mathbf{h} \cdot \mathbf{m}\, dx.$$

Example (Exchange length and typical time scale for Permalloy)

For Permalloy, $A = 1.3 \times 10^{-11}\,\mathrm{J/m}$, $M_s = 0.69 \times 10^6\,\mathrm{A/m}$. We find a typical length scale

$$\ell_{\mathrm{ex}} = 6.59\,\mathrm{nm}.$$

We find a typical time scale ($\gamma = 1.76 \times 10^{11}\,s^{-1}\,T^{-1}$, $\mu_0 = 4\pi \times 10^{-7}\,N\,A^{-2}$ and $\mathrm{T} = \mathrm{N}/(\mathrm{A}\cdot\mathrm{m})$)

$$\frac{1}{\gamma\mu_0 M_s} = 6.5 \times 10^{-12}\,\mathrm{sec} = 6.55\,\mathrm{psec}.$$

A corresponding frequency is

$$\gamma\mu_0 M_s = 150\,\mathrm{GHz}.$$

Lecture 3b. Quiz. Extra anisotropy in the equation.

Suppose a configuration $\mathbf{m} = \mathbf{m}(x)\hat{\boldsymbol{e}}_1$. See in the notes the magnetostatic field that this creates.

1. Write the Landau-Lifshitz equation (with an exchange, easy-axis anisotropy, magnetostatic field).
2. We have seen a domain wall solution

$$m_1 = \operatorname{sech}(x)\cos(\phi_0), m_2 = \operatorname{sech}(x)\sin(\phi_0), m3 = \tanh(x).$$

For which ϕ_0 is this a solution of the model including the magnetostatic field?

Lecture 3b. Model for an infinitely thick film (1D)

We have seen that a configuration with $m_1(x) \neq 0$ would generate a magnetostatic field $\mathbf{h}_m = -m_1(x)\,\hat{\boldsymbol{e}}_1$.
We expand our model to include such a magnetostatic field term.

An infinitely thick film

$$\frac{\partial \mathbf{m}}{\partial \tau} = -\mathbf{m} \times \left(\mathbf{m}'' + k^2\, m_3 \hat{\mathbf{e}}_3 - m_1 \hat{\mathbf{e}}_1\right)$$

- The magnetostatic field is modelled as anisotropy with a hard axis in the $\hat{\boldsymbol{e}}_1$ direction (i.e., easy-plane yz).
- The model now contains two anisotropy terms.

The domain wall is

$$m_1 = \operatorname{sech}(kx) \cos\phi_0, \quad m_2 = \operatorname{sech}(kx) \sin\phi_0, \quad m_3 = \tanh(kx)$$

It is a solution of the model for $\phi_0 = \pm\pi/2$ (Bloch wall), i.e., $m_1 = 0$.

Traveling waves

The Landau-Lifshitz equation

$$\frac{\partial \mathbf{m}}{\partial \tau} = -\mathbf{m} \times \left(\partial_x^2 \mathbf{m} + k^2\, m_3 \hat{\boldsymbol{e}}_3 - k_1^2\, m_1 \hat{\boldsymbol{e}}_1 \right)$$

We added a parameter k_1 in the new term only to trace its effect.

Traveling wave ansatz

Assume a wave propagating with velocity v. That is, we make the ansatz $\mathbf{m}(x,\tau) \to \mathbf{m}(x - v\tau)$ for which $\partial_x = \mathbf{m}'$, $\partial_\tau \mathbf{m} = -v\mathbf{m}$. The LL equation becomes

$$v\mathbf{m}' = \mathbf{m} \times \left(\mathbf{m}'' + k^2\, m_3 \hat{\boldsymbol{e}}_3 - k_1^2\, m_1 \hat{\boldsymbol{e}}_1 \right).$$

Traveling domain walls

Propagating domain wall solutions with velocity v

The propagating domain wall

$$m_1 = \operatorname{sech}(\epsilon\xi)\cos\phi_0, \quad m_2 = \operatorname{sech}(\epsilon\xi)\sin\phi_0, \quad m_3 = \tanh(\epsilon\xi)$$

where $\xi = x - vt$, is a solution of the equation.

- The parameter ϕ_0 is the "wall tilting" (f0r $\phi_0 \neq \pm\pi/2$ the tilting is out of the plane of the wall).
- The parameter ϵ gives the domain wall width, $\delta = 1/\epsilon$.

Remark

A geodesic (meridian) on the sphere is connecting the poles $m_3 = \pm 1$.

Traveling domain walls: Walker solution

Propagating domain wall solutions

The equation is satisfied only when the parameters are related by

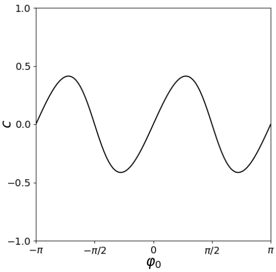
$$v = -\frac{\sin(2\varphi_0)}{2\epsilon}, \quad \epsilon = \pm\sqrt{k^2 + \cos^2\varphi_0}.$$

We have a family of propagating domain walls with $|v| \leq 1/(2\epsilon)$.

- Choose some tilting ϕ_0.
- Calculate the parameter ϵ ($|\epsilon| > k$).
- We now have the velocity of the wall.

- Static walls ($v = 0$)
  Bloch wall for $\varphi_0 = \pm\pi/2$
  Nèel wall for $\varphi_0 = 0, \pi$.
- Maximum velocity for some $\pi/4 < \phi_0 < \pi/2$.

Velocity vs tilting angle



Traveling domain walls: Walker solution (sketch)

Traveling domain walls: Walker solution (role of k_1)

Propagating domain wall solutions

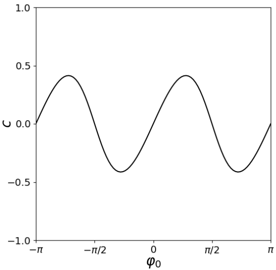
The equation is satisfied only when the parameters are related by

$$v = -k_1^2 \frac{\sin(2\varphi_0)}{2\epsilon}, \quad \epsilon = \pm\sqrt{k^2 + k_1^2 \cos^2\varphi_0}.$$

We have a family of propagating domain walls with $|v| \leq 1/(2\epsilon)$.

- Choose some tilting ϕ_0.
- Calculate the parameter ϵ (note $|\epsilon| \geq k$).
- We now have the velocity of the wall.

Velocity vs tilting angle



- Static walls ($v = 0$)
  Bloch wall for $\varphi_0 = \pm\pi/2$
  Nèel wall for $\varphi_0 = 0, \pi$.
- Maximum velocity for some $\pi/4 < \phi_0 < \pi/2$.

There are no propagating walls for $k_1 = 0$.

Dzyaloshinskii-Moriya (DM) materials

Chiral domain walls

Add a chiral (Dzyaloshinskii-Moriya) term in the equation

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times \left(\partial_x^2 \mathbf{m} + k^2\, m_3 \hat{\boldsymbol{e}}_3 - 2\lambda\, \hat{\boldsymbol{e}}_1 \times \partial_x \mathbf{m} \right).$$

Static domain wall

For a Bloch wall ($m_1 = 0$) the DM term is identically zero, i.e., it is a solution of the model including DM interaction (DMI).

Dynamical wall solutions

- Propagating domain walls exist in this model.
- Some issues on the problem of dynamics are open for chiral domain walls.