Outline

Magnetic domain walls

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- propagating walls

Magnetic solitons on the plane

- vortices
- topological number
- magnetic bubbles

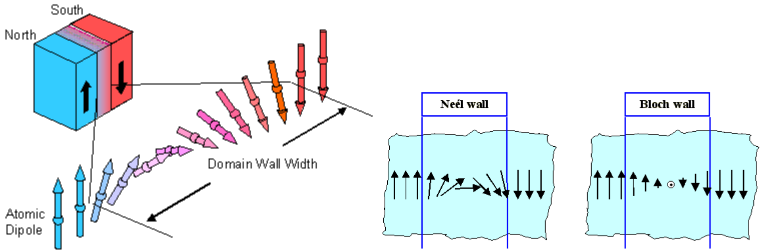
Magnetic domain wall

Consider a bulk ferromagnet which is magnetized “up” ($\boldsymbol{M} = M_s\hat{\boldsymbol{z}}$) on one end, and “down” ($\boldsymbol{M} = -M_s\hat{\boldsymbol{z}}$) on its other end. A domain wall exists between the two domains. [Landau and Lifshitz (1935)]:

$$m_z = \tanh(x\sqrt{Q}) \Longrightarrow M_z = M_s \tanh(x/\sqrt{K/A}),$$
$$m_y = 1/\cosh(x\sqrt{Q}) \Longrightarrow M_y = M_s/\cosh(x/\sqrt{K/A}).$$

(We assume only exchange and easy-axis anisotropy.)
This is a solution of the Landau-Lifshitz equation.
The domain wall width is $\delta = \sqrt{K/A}$.
This is called a Bloch wall.

Sketches for domain walls



Find the magnetostatic field of a Bloch wall:

$$\nabla \boldsymbol{h} = -\nabla \boldsymbol{m}, \quad \nabla \times \boldsymbol{h} = 0 \quad \Longrightarrow \quad \boldsymbol{h} = 0.$$

$$\nabla \boldsymbol{m} = \frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} + \frac{\partial m_z}{\partial z} = 0.$$

Dissipation of the magnetisation dynamics

We add a phenomenological dissipation term in the Landau-Lifshitz equation. This is refered to as Gilbert damping. Here is the Landau-Lifshitz-Gilbert equation:

$$\frac{\partial \boldsymbol{m}}{\partial t} - \alpha \left(\boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t} \right) = -\boldsymbol{m} \times \boldsymbol{f}.$$

α: a dimensionless damping constant (typically $\alpha \sim 0.02$).

Note that this equation conserves the length of $\boldsymbol{m}$. This is easier to see in its alternative form:

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\alpha_1 (\boldsymbol{m} \times \boldsymbol{f}) - \alpha_2 [\boldsymbol{m} \times (\boldsymbol{m} \times \boldsymbol{f})], \quad \alpha_1 = \frac{1}{1+\alpha^2}, \; \alpha_2 = \frac{\alpha}{1+\alpha^2}.$$

To be sure, the energy is continuously decreasing:

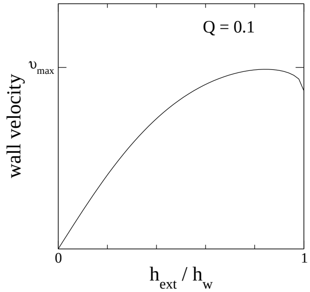
$$\frac{dE}{dt} = \ldots = -\alpha_2 \int \left(\frac{\partial \boldsymbol{m}}{\partial t} \right)^2 dV < 0.$$

Propagating wall

We suppose a uniform external magnetic field $\boldsymbol{h}_{\rm ext} = (0, 0, h_{\rm ext})$. We find a solution of the Landau-Lifshitz-Gilbert equation of the form $\boldsymbol{m} = \boldsymbol{m}(x - vt)$. The Bloch wall has to tilt by a constant angle Φ. We then find:

$$v = \frac{h_{\rm ext}}{\varepsilon\,\alpha} \quad \text{and} \quad v = -\frac{\sin(2\Phi)}{2\varepsilon}, \qquad \varepsilon \equiv Q + \cos^2\Phi$$

$$\Rightarrow h_{\rm ext} = -\frac{\alpha}{2}\sin(2\Phi), \quad \text{which means that} \quad |h_{\rm ext}| \le \frac{\alpha}{2}.$$



The maximum field $h_w \equiv \alpha/2$ is the "Walker field".

Correspondingly:

$$v_{\rm max} = \frac{1}{2\sqrt{Q+1/2}}$$

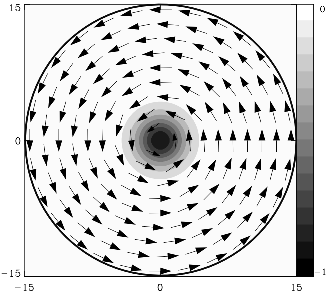
is the "Walker velocity".

Vortices

Consider a very thin disc element, and suppose that the magnetisation vector lies on the plane.

Also assume that it is tangential to the lateral particle surface.

At the particle center, $\boldsymbol{m}$ cannot be on the plane, it has to be "out-of-plane", i.e., $m_z = \pm 1$.



Note: it is not difficult to find the vortex profile by solving numerically the Landau-Lifshitz-Gilbert equation.

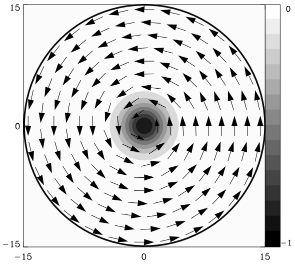
The general axially symmetric vortex on the plane

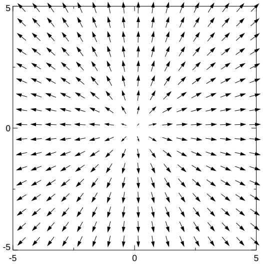
Parametrise the magnetisation vector by the angles Θ, Φ.

$$\begin{aligned}\Theta &= \theta(\rho), \qquad \theta(\rho=0)=0,\pi \ \ [\Rightarrow m_z(\rho=0)=\pm 1],\\ \Phi &= \kappa(\phi+\phi_0), \qquad \kappa=\pm 1, \quad \phi_0 : \text{const.}\end{aligned}$$

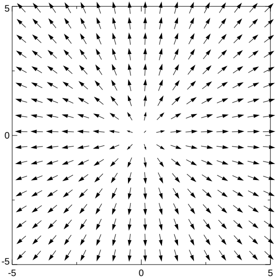
$m_z(\rho = 0)$: polarity (or magnetisation).

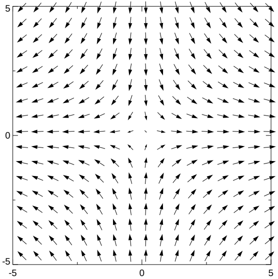
ϕ_0: phase (or orientation).

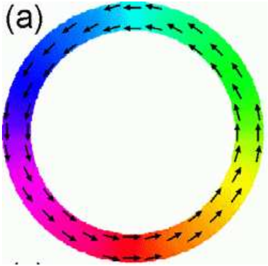


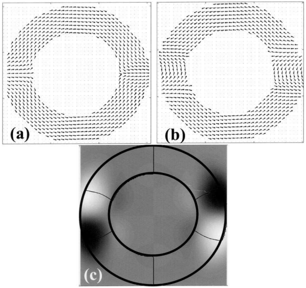


κ: vortex number.





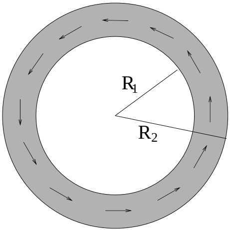




[Castano, Ross, et al, PRB **67**, 184425 (2003).]

Vortex in a ring element

$$m_1 = \cos(\phi + \phi_0),$$
$$m_2 = \sin(\phi + \phi_0),$$
$$m_3 = 0. \qquad \phi_0 = \pi/2.$$



Exchange field:

$$\Delta \boldsymbol{m} = \frac{1}{\rho^2}\frac{\partial^2 m_1}{\partial \phi^2}\hat{\boldsymbol{x}} + \frac{1}{\rho^2}\frac{\partial^2 m_2}{\partial \phi^2}\hat{\boldsymbol{y}} = \ldots = -\frac{\boldsymbol{m}}{\rho^2} \Rightarrow \boldsymbol{m} \times \Delta \boldsymbol{m} = 0.$$

Magnetostatic field:

$$\boldsymbol{\nabla} \cdot \boldsymbol{m} = \ldots = \frac{1}{\rho}\cos\phi_0.$$

For $\phi_0/2 = 0$ we have $\boldsymbol{\nabla} \cdot \boldsymbol{m} = 0$ and, since we have no surface charges $\Longrightarrow \boldsymbol{h} = 0$.

Energy

$$\begin{aligned} E &= E_{\mathrm{ex}} = \frac{1}{2} \int \left\{ \left(\frac{\partial \boldsymbol{m}}{\partial \rho} \right)^2 + \left(\frac{\partial \boldsymbol{m}}{\partial z} \right)^2 + \frac{1}{\rho^2} \left(\frac{\partial \boldsymbol{m}}{\partial \phi} \right)^2 \right\} dV \\ &= \ldots = \pi\, t \ln \left(\frac{R_2}{R_1} \right). \end{aligned}$$

Note: The vortex with $\kappa = -1$ has the same exchange energy, but it has a nonvanishing magnetostatic energy.

Topological numbers

On a circle

Suppose a thin film (two-dimensional material). Run around a circle at spatial infinity and follow the vector $\boldsymbol{m}$ as it rotates.

E.g., a vortex has topological number equal to $\kappa = 1$.
Also possible are $\kappa = 0, \pm 1, \pm 2 \ldots$.

On a sphere

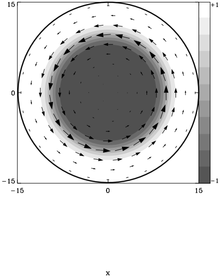
Suppose that as $\boldsymbol{r} \to \infty$, $\boldsymbol{m} \to$ *constant vector* (e.g., $\boldsymbol{m} \to \hat{\boldsymbol{z}}$).
Run over the plane and follow $\boldsymbol{m}$ as it runs on the sphere.

E.g., a vortex has topological number (winding number) equal to $\mathcal{N} = 1/2$.
Also possible are $\mathcal{N} = 0, \pm 1, \pm 2 \ldots$.

Now, how do we obtain $\mathcal{N} = 1$?

Magnetic bubbles

Consider a continuous film with a strong perpendicular anisotropy, so that $\boldsymbol{m} = \hat{\boldsymbol{z}}$ at $\boldsymbol{r} = \infty$.



The winding number is given by

$$\mathcal{N} = \frac{1}{4\pi} \int n \, d^2x, \qquad n = \frac{1}{2} \epsilon_{\mu\nu} \left(\partial_\nu \boldsymbol{m} \times \partial_\mu \boldsymbol{m} \right) \cdot \boldsymbol{m}.$$

In the case of axial symmetry this is simplified to

$$\mathcal{N} = \frac{1}{2} \left[m_3(\rho = \infty) - m_3(\rho = 0) \right],$$

which easily gives $\mathcal{N} = 1$ for the fundamental bubble.

Magnetic bubbles

... but we could also imagine bubbles with different winding numbers: